Math 1552 Summer 2023
Quiz 4 June 15
Time limit: 20 Minutes

Name (Print): Key

Teaching Assistant/Section: ________

GT ID: | | | | | | | | | |

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Sign Your Name: Jal

Please clearly organize your work, show all steps, simplify all answers, and $\boxed{\text{BOX}}$ your answers.

1. (4 points) Find a general formula a_n for the n-th term of the sequence. You do not need to show work on this problem but please put your final answer in the box.

   Hint: be sure to include your starting value for n.

$$\frac{1}{3}, \frac{2}{6}, \frac{2^2}{9}, \frac{2^3}{12}, \frac{2^4}{15}, \ldots$$

numerators are powers of 2

denominators are multiples of 3

$$\boxed{\frac{2^n}{3(n+1)},\ n \geq 0}$$

2. (10 points) Evaluate the improper integral.

$$\int_2^\infty \frac{2x}{(x^2+1)^2}\,dt$$

$$\boxed{\begin{aligned} u &= x^2+1 \\ du &= 2x\,dx \end{aligned}}$$

$$\int_2^N \frac{2x}{(x^2+1)^2}\,dx = \int_*^* \frac{1}{u^2}\,du = \left.\frac{-1}{u}\right|_*^*$$

$$= \left.\frac{-1}{x^2+1}\right|_2^N = \frac{-1}{N^2+1} - \frac{-1}{5} = \frac{1}{5} - \frac{1}{N^2+1}$$

So $\displaystyle\int_2^\infty \frac{2x}{(x^2+1)^2}\,dx = \lim_{N\to\infty} \frac{1}{5} - \frac{1}{N^2+1} = \frac{1}{5} - 0 = \boxed{\frac{1}{5}}$

3. (6 points) For each sequence, determine the limit of the sequence as n tends to infinity. If the limit diverges, write either DNE, ∞ DNE, or $-\infty$ DNE in the box, as appropriate. You do not have to show your work for problems on this page, but please put your final answer in the box.

(a) $\left\{\left(1-\frac{2}{n}\right)^n\right\}$

$\boxed{e^{-2}}$

$a_n = \left(1-\frac{2}{n}\right)^n$

$b_n = \ln(a_n) = n \cdot \ln\left(1-\frac{2}{n}\right) = \frac{\ln\left(1-\frac{2}{n}\right)}{1/n}$

$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{\frac{1}{1-2/n}\cdot\left(1-\frac{2}{n}\right)'}{-1/n^2} = \lim_{n\to\infty} \frac{1}{1-2/n}\cdot\frac{-2}{\cancel{n^2}}\cdot -\cancel{n^2} = -2$

L'Hop "$\frac{0}{0}$"

Since $a_n = e^{b_n} = e^{\ln(a_n)}$ and $b_n \to -2$, then $a_n \to e^{-2}$

(b) $\left\{\frac{3^n}{n!}\right\}$

$\boxed{0}$

Notice that $a_n = \frac{3^n}{n!}$ satisfies

$a_{n+1} < \frac{1}{2} a_n$ whenever $n \geq 6$, since

$a_{n+1} = \frac{3^{n+1}}{(n+1)!} = \frac{3\cdot 3^n}{(n+1)\,n!} = \frac{3}{n+1}\cdot a_n$. So $a_n \to 0$

(c) $\left\{(-1)^n \frac{n}{n+1}\right\}$

$\boxed{\text{DNE}}$

Since $a_n = \frac{n}{n+1}$ converges to 1,

the alternating sequence $(-1)^n \frac{n}{n+1}$ diverges