Math 1552 Summer 2023
Quiz 5 July 6
Time limit: 20 Minutes

Name (Print): Key

Teaching Assistant/Section:

GT ID: | | | | | | | | | |

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Sign Your Name: Sal

Please clearly organize your work, show all steps, simplify all answers, and $\boxed{\text{BOX}}$ your answers.

1. (3 points) For the given series $\sum a_n$, write the ratio $\frac{a_{n+1}}{a_n}$ from the ratio test. Simplify your answer but *do not* take a limit.

$$\sum_{n=1}^{\infty} \frac{3^n}{(n+1)!}$$

$$a_n = \frac{3^n}{(n+1)!}$$

$$a_{n+1} = \frac{3^{n+1}}{(n+1+1)!} = \frac{3 \cdot 3^n}{(n+2)!}$$

$$\frac{a_{n+1}}{a_n} = \frac{3\cdot\cancel{3^n}}{(n+2)!} \cdot \frac{(n+1)!}{\cancel{3^n}}$$

$$= \frac{3}{n+2}$$

$$\frac{a_{n+1}}{a_n} = \boxed{3/n+2}$$

2. (3 points) Briefly explain the flaw in the following argument. Use complete sentences, justify your reasoning, and use correct terminology from the class.

The series $\sum_{n=1}^{\infty} \frac{n^2+1}{n^4+1}$ converges by the direct comparison test, since you can compare the terms $a_n = \frac{n^2+1}{n^4+1}$ to the terms $b_n = \frac{n^2}{n^4} = \frac{1}{n^2}$.

Note: $a_n = \frac{n^2+1}{n^4+1} \nleq \frac{n^2}{n^4} = b_n$ For example, when $n=2$ $a_2 = \frac{5}{17}$ and $b_2 = \frac{4}{16} = \frac{1}{4}$ but $5/17 \nleq 1/4$

Since $a_n \nleq b_n$ even though Σb_n converges you can <u>NOT</u> conclude that Σa_n converges.

3. (points) Determine if each series converges or diverges. Fully justify your answer for credit, *e.g.*, state the convergence test you used and clearly state the necessary conditions for the test you are using. Points will be deducted for arguments that are not clearly organized.

(a) $\sum_{n=1}^{\infty} \frac{n^2+1}{n^4+1}$ limit comparison w/ $b_n = 1/n^2$

$a_n = \frac{n^2+1}{n^4+1}$ $b_n = 1/n^2$

$\frac{a_n}{b_n} = \frac{n^2+1}{n^4+1} \cdot \frac{n^2}{1} = \frac{n^4+n^2}{n^4+1} \xrightarrow[n\to\infty]{} 1 = C$

Since $C>0$, and finite, by limit comparison test

$\Rightarrow$ $\sum a_n$ & $\sum b_n$ both converge or both diverge.

But $\sum b_n = \sum \frac{1}{n^2}$ converges by p-series test w/ $p = 2 > 1$

So $\boxed{\sum a_n \text{ converges}}$

(b) $\sum_{n=1}^{\infty} \frac{4^n}{(3n)^n}$

root test

$a_n = \frac{4^n}{(3n)^n}$

So

$(a_n)^{1/n} = \left(\frac{4^n}{(3n)^n}\right)^{1/n} = \frac{4}{3n} \to 0 = L$

Since $L<1$, by the root test

$\boxed{\sum a_n \text{ converges}}$