Math 1552 Summer 2023
Quiz 5 Practice July 6

Name (Print): Key

Teaching Assistant/Section:

By signing here, you agree to abide by the **Georgia Tech Honor Code**: *I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech Community.*

Sign Your Name: Joel

Please clearly organize your work, show all steps, simplify all answers, and $\boxed{\text{BOX}}$ your answers.

1. (3 points) For the given series $\sum a_n$, write the ratio $\frac{a_{n+1}}{a_n}$ from the ratio test. Simplify your answer but *do not* take a limit.

$$\sum_{n=1}^{\infty} \frac{n!}{3n^2}$$

$$a_n = \frac{n!}{3n^2} \qquad \frac{a_{n+1}}{a_n} = \frac{(n+1)!}{\cancel{3}(n+1)^2} \cdot \frac{\cancel{3}n^2}{n!} = \frac{n^2(n+1)}{(n+1)^{\cancel{2}}} = \frac{n^2}{n+1} \qquad \frac{a_{n+1}}{a_n} = \boxed{n^2/n+1}$$

$$a_{n+1} = \frac{(n+1)!}{3(n+1)^2}$$

2. (3 points) Briefly explain the flaw in the following argument. Use complete sentences, justify your reasoning, and use correct terminology from the class.

The series $\sum_{n=2}^{\infty} \frac{\sqrt{n}}{\sqrt{n^3-1}}$ converges by the direct comparison test, since you can compare the terms $a_n = \frac{\sqrt{n}}{\sqrt{n^3-1}}$ to the terms $b_n = \frac{\sqrt{n}}{\sqrt{n^3}} = \frac{1}{n}$.

false

$$a_n = \frac{\sqrt{n}}{\sqrt{n^3-1}} \geq \frac{\sqrt{n}}{\sqrt{n^3}} = \left(\frac{n}{n^3}\right)^{1/2} = \frac{1}{n} = b_n$$

So you can compare a_n to b_n, but $a_n \geq b_n$
So a_n is BIGGER than b_n.

Also $\sum b_n$ diverges by p-test w/ $p=1$, so in fact since $\sum a_n \geq \sum b_n$ the series $\sum a_n$ actually <u>diverges</u>, not <u>converges</u>.

3. (8 points) Determine if each series converges or diverges. Fully justify your answer for credit, *e.g.*, state the convergence test you used and clearly state the necessary conditions for the test you are using. Points will be deducted for arguments that are not clearly organized.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{\sqrt{n}}{\sqrt{n^3-1}}$ limit comparison test

$a_n = \frac{\sqrt{n}}{\sqrt{n^3-1}}$ $b_n = 1/n$

$\frac{a_n}{b_n} = \frac{\sqrt{n}}{\sqrt{n^3-1}} \cdot \frac{n}{1} = \frac{\sqrt{n}\cdot\sqrt{n^2}}{\sqrt{n^3-1}} = \sqrt{\frac{n^3}{n^3-1}}$

So

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \sqrt{\frac{n^3}{n^3-1}} = \sqrt{1} = 1 = c$

Since $c>0$, by limit comparison test Σa_n & Σb_n both converge or both diverge.

Since $\Sigma b_n = \Sigma \frac{1}{n}$ diverges (p-series test w/ $p=1$)

The series $\boxed{\Sigma a_n \text{ also diverges}}$

(b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{4n+3}{3n-5}\right)^n$

root test

$a_n = \left(\frac{4n+3}{3n-5}\right)^n$

$(a_n)^{1/n} = \left[\left(\frac{4n+3}{3n-5}\right)^n\right]^{1/n} = \frac{4n+3}{3n-5}$

So

$\lim_{n\to\infty} (a_n)^{1/n} = \lim_{n\to\infty} \frac{4n+3}{3n-5} = \frac{4}{3} = L$

Since $L>1$, the series $\boxed{\Sigma a_n \text{ diverges}}$ by the root test