# Section 1.7: Linear Independence

## Quick summary

A set of vectors $\{v_1, \ldots, v_k\}$ in $\mathbb{R}^n$ is linearly independent if the vector equation

$$c_1 v_1 + c_2 v_2 + \cdots + c_k v_k = 0 \ldots$$

The columns of an $m \times n$ matrix $A$ are linearly independent $\Leftrightarrow$ $A$ has...

To check if $\{v_1, \ldots, v_k\}$ is linearly independent check if each $v_i$ is...

*Fact.* Say $v_1, \ldots, v_k$ are in $\mathbb{R}^n$. If $k > n$, then $\{v_1, \ldots, v_k\}$ is

# Section 1.7: Linear Independence

## Concept questions

Q1. *True/False.* If three vectors span $\mathbb{R}^3$ then those three vectors must be linearly independent.

Q2. Which of the following true statements can be checked without row reduction?

1. $\{(3,3,4),(0,0,\pi),(0,\sqrt{2},0)\}$ is linearly independent
2. $\{(3,3,4),(0,10,20),(0,5,7)\}$ is linearly independent
3. $\{(3,3,4),(0,10,20),(0,5,7),(0,0,1)\}$ is linearly dependent
4. $\{(3,3,4),(0,10,20),(0,0,0)\}$ is linearly dependent

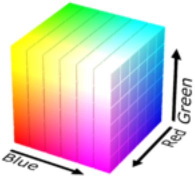
Q3. How many solutions can there be to $Ax = b$ if the columns of $A$ are linearly independent?

1. 0
2. 1
3. $\infty$

# Section 1.7: Linear Independence

## Application: Additive Color Theory

Every color is a vector in $\mathbb{R}^3$ with coordinates between 0 and 256. The three coordinates correspond to red, green, and blue.

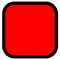
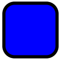
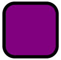



Given colors $v_1, \ldots, v_k$, we can form a new color by making a linear combination

$$c_1 v_1 + \cdots + c_k v_k$$

where $c_1 + \cdots + c_k = 1$

Example:

$\frac{1}{2}$  $+$ $\frac{1}{2}$  $=$ 

# Section 1.7: Linear Independence

## Application: Additive Color Theory

Consider now the three colors

$$\begin{pmatrix} 240 \\ 140 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 120 \\ 100 \end{pmatrix}, \begin{pmatrix} 60 \\ 125 \\ 75 \end{pmatrix}$$

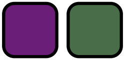
Are these colors linearly independent? What does your answer tell you about the colors?

# Section 1.7: Linear Independence

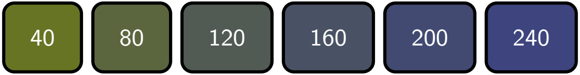
## Application: Additive Color Theory

Consider now the two colors

$$\begin{pmatrix} 180 \\ 50 \\ 200 \end{pmatrix}, \begin{pmatrix} 100 \\ 150 \\ 100 \end{pmatrix}$$



For which $h$ is $(116, 130, h)$ in the span of those two colors?



Given an $m \times n$ matrix $A$ we define a function

$$T_A : \mathbb{R}^n \to \mathbb{R}^m$$
$$T_A(v) =$$

The domain of $T_A$ is
The co-domain/target of $T_A$ is
The range/image of $T_A$ is

When $m = n$ we can think of $T_A$ as doing something to $\mathbb{R}^n$.

A function $T : \mathbb{R}^n \to \mathbb{R}^m$ is linear if

- $T(u + v) =$
- $T(cv) =$

*Fact.* Every matrix transformation $T_A$ is linear.

# Section 1.8: Linear Transformations

## Concept Questions

Q1. Say $A$ is a $1 \times n$ matrix. If $T_A(v) = 3$ and $T_A(w) = -1$, then what is

$$T_A(7v - 5w)?$$

Q2. Find a $3 \times 3$ matrix $A$ so that $T_A(v) = v$ for all $v$ in $\mathbb{R}^3$.

Q3. Say $A$ is an $m \times 2$ matrix. If the columns of $A$ are linearly independent, what does the image of $T_A$ look like geometrically?

For each matrix $A$, describe what $T_A$ does to $\mathbb{R}^3$.

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$