Math 1553: Intro to Linear Algebra

Chapter 1 Review for Test #1

1. **Terminology Review**

(a) A matrix is in RREF if the matrix satsifies the following four conditions:

1.
2.
3.
4.

(b) The three allowable row operations are:

1.
2.
3.

(c) If the system of equations has infinitely many solutions, we paramaterize the solution by setting each _____ variable equal to a parameter, and solve for the variables in _____ columns. The number of parameters is equal to the number of _____ columns.

(d) A linear combination of the vectors $\{\vec{v_1}, \vec{v_2}, ..., \vec{v_k}\}$ is defined as:

(e) If a set of vectors is linearly independent, then no vector is a _____ _____ of the other vectors in the set. To determine the number of linearly independent vectors, we row reduce and count the number of _____ columns.

(f) The span of a set of vectors is the set of all _____ _____ of the vectors. In $\Re^3$, the span must be either: _____, _____, _____, or _____.

(g) If $T : \Re^n \to \Re^m$ is a linear transformation, then the matrix A of T has dimension _____. The domain of T is _____ and the codomain of T is _____.

(h) Finding the range of a transformation is equivalent to finding the _____ of the columns of A.

(i) If a linear transformation is 1-to-1, then the row-reduced matrix must have a pivotal one in every _____. If the tranformation is onto, then the row-reduced matrix has a pivotal one in every _____.

2. Determine whether each of the following augmented matrices are in reduced row echelon form (RREF). If so, write out the solution. If not, explain what would need to change in order to obtain a matrix in RREF.

(a) $\left[\begin{array}{ccccc|c} 1 & 2 & 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 5 \\ 0 & 0 & 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$

(b) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 3 & 6 \\ 0 & 0 & 1 & 4 \end{array}\right]$

(c) $\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 2 & 8 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 2 \end{array}\right]$

(d) $\left[\begin{array}{ccccc|c} 1 & 0 & 1 & 0 & 5 & 3 \\ 0 & 1 & -1 & 0 & 4 & 9 \\ 0 & 0 & 0 & 1 & 3 & 7 \end{array}\right]$

3. Let $f : R^n \to R^m$ be a linear transformation. Determine if the following statements are always true or sometimes false. If the statement is sometimes false, explain why.

(a) f maps the zero vector to the zero vector.

(b) The standard matrix of f has dimension $n \times m$.

(c) Since f is linear, $f(a\mathbf{x}+b\mathbf{y}) = af(\mathbf{x})+bf(\mathbf{y})$.

(d) The second row of the standard matrix of f represents the image of $\mathbf{e}_2$ under f.

4. Find the standard matrix for the following transformation from $\Re^2$ to $\Re^2$: a dilation with a factor of $k = 2$, followed by a rotation of $45°$ in the counterclockwise direction, followed by a reflection about the y-axis.

5. (a) Find all values of a_1, a_2, and a_3 so that

$$a_1 \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix} + a_2 \begin{bmatrix} 4 \\ 0 \\ -8 \end{bmatrix} + a_3 \begin{bmatrix} 6 \\ -1 \\ -4 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix}.$$

(b) Do the four vectors in part (a) form a linearly independent set? If not, which subsets of these four vectors will form a linearly independent set?

(c) Describe the span of the four vectors in part (a).

6. Use the Gauss-Jordan elimination method to solve the following system of equations. You should continue your row operations until your obtain a matrix in RREF. Describe your answer geometrically (is it a point, line, plane, hyperplane, etc?).

$$\begin{cases} x_1 + 3x_2 + 2x_3 + 2x_5 = 0 \\ 2x_1 + 6x_2 - 5x_3 - 2x_4 + 4x_5 - 3x_6 = -1 \\ 5x_3 + 10x_4 + 15x_6 = 5 \\ 2x_1 + 6x_2 + 8x_4 + 4x_5 + 18x_6 = 6 \end{cases}$$

7. The second little pig has decided to build his house out of sticks. His house is shaped like a pyramid with a triangular base that has vertices at the points (0,0,0), (2,0,0), (0,2,0), and (1,1,1). The big bad wolf finds the pig's house and blows it down so that the house is rotated by an angle of 45° in a counterclockwise direction about the z-axis, and then projected onto the xy-plane. Find a matrix A to represent the transformation that destroys the house.