Math 1553, Intro to Linear Algebra
Sections 1.1-1.2: Gauss-Jordan Elimination

Name:

1. Determine whether each of the following augmented matrices are in reduced row echelon form (RREF). If so, write the solution. If not, use row operations to change the matrix to RREF and then state the solution.

(a) $\left[\begin{array}{ccccc|c} 1 & 2 & 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 5 \\ 0 & 0 & 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$

(b) $\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 2 & 8 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 2 \end{array}\right]$

2. Are the following statements always true or sometimes false? If the statement is sometimes false, explain why.

(a) If a matrix is reduced to RREF by two different sequences of row operations, then the resulting matrices will be different.

(b) If the RREF form of a matrix has a row of zeros, then the system has infinitely many solutions.

(c) If there are more unknowns than equations, then the system cannot have a unique solution.

(d) If there are more equations than unknowns, then the system cannot have a unique solution.

(e) If a linear system of n equations in n unknowns has RREF form with n leading 1's, then the system has a unique solution.

3. Use the Gauss-Jordan elimination method to solve the following system of equations, or explain why a solution does not exist. You should continue the row operations until you obtain a matrix in RREF.

$$\begin{cases} x_1 + x_2 + x_3 - x_4 & = -3 \\ 2x_1 + 3x_2 + x_3 - 5x_4 & = -9 \\ x_1 + 3x_2 - x_3 - 6x_4 & = 7 \end{cases}$$