Math 1553: Intro to Linear Algebra

Sections 1.4, 1.5, 1.7: Linear Independence and Vector Equations

Name: _____

Your professor decides to give three different grading schemes for computing your final course average. The scheme is based on homework (HW), quiz grades (Q), midterms (M), and a final exam (F). The three schemes provided can be described in the following matrix:

$$\begin{array}{c} \\ Scheme1 \\ Scheme2 \\ Scheme3 \end{array} \begin{bmatrix} HW & Q & M & F \\ 0.10 & 0.10 & 0.50 & 0.30 \\ 0.10 & 0.10 & 0.40 & 0.40 \\ 0.10 & 0.10 & 0.60 & 0.20 \end{bmatrix}.$$

1. If the student has a score of x_1 on HW, x_2 on quizzes, x_3 on midterms, and x_4 on the final, with possible final averages of b_1, b_2, b_3, write a matrix equation $A\vec{x} = \vec{b}$ to determine the final averages under the three schemes.

2. Suppose a student has a 90 homework average, 90 quiz average, 85 midterm average, and a 95 on the final exam. Which scheme would give the student the highest overall grade? USE MATRIX MULTIPLICATION.

3. (a) Keeping the vectors $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ as general vectors, write down the augmented matrix $[A|\vec{b}]$.

(b) Row reduce the augmented matrix from part (a) until you reach row echelon form (REF).

(c) Looking at the final matrix in part (b), what equation in terms of the coordinates of $\vec{b}$ must be satisfied in order to have a solution?

(d) Your answer to part (c) describes the span of the columns of A. Describe the span geometrically.

(e) Solve the equation in part (c) for b_1. Looking at this equation, is it possible for b_1 to be the largest value of b_1, b_2, b_3? In other words: is it ever possible for the grade under Scheme 1 to be the highest of the three averages? Why or why not?