Math 1553 **Exam 1** Fall 2016

Instructor: Sal Barone (B)

Name: _____

GT username: _____

1. No books or notes are allowed.
2. All calculators and/or electronic devices are not allowed.
3. Show all work and fully justify your answer to receive full credit.
4. Please BOX your answers.
5. Good luck!

Page	Max. Possible	Points
1	15	
2	20	
3	17	
4	18	
5	30	
Total	100	

1. Determine whether $\begin{bmatrix} 2 \\ -2 \\ -3 \end{bmatrix}$ belongs to the span of the vectors $\begin{bmatrix} 1 \\ -3 \\ -2 \end{bmatrix}$, $\begin{bmatrix} 3 \\ -9 \\ -6 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$.

Justify your answer for full credit. (15 pts.)

2. Solve the system of linear equations.

$$\begin{aligned} 2x + 2y \quad &+z = 2 \\ x + y \quad &= 0 \\ 2x + y \quad &+z = 1 \\ 3x + 2y \quad &+z = 1 \end{aligned}$$

(15 pts.)

3. If $Ax = b$ has a unique solution, does that imply that the columns of A are linearly independent? Justify your answer for full credit. (3 pts. answer, 2 pts justification)

4. Suppose A is row equivalent to

$$\begin{bmatrix} 1 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -4 & -4 & 0 & 8 & 1 \end{bmatrix}$$

(a) Describe the solutions to $A\mathbf{x} = 0$ in parametric vector form. (10 pts.)

(b) Describe the set of solutions to $A\mathbf{x} = 0$ geometrically in a few words. (4 pts.)

(c) Is $\mathbf{x} = \begin{bmatrix} 4 \\ 2 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ a solution to $A\mathbf{x} = 0$? Justify your answer. (3 pts.)

5. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation which first rotates the vectors in $\mathbb{R}^2$ counter-clockwise by $45\,^\circ$, and then reflects the resulting vector across the vertical y-axis.

(a) Find the standard matrix T. (10 pts.)

(b) Is T one-to-one? Explain for full credit. (3 pts. for ans., 2 pts. for just.)

(c) Is T onto? Explain for full credit. (2 pts. for answer, 1 pt. for justification)

6. Determine whether or not the vectors $\begin{bmatrix}1\\0\\2\end{bmatrix}$, $\begin{bmatrix}2\\1\\0\end{bmatrix}$, $\begin{bmatrix}-1\\-2\\6\end{bmatrix}$ are linearly independent. Fully justify your answer for full credit. (15 pts.)

7. For each 3×3 matrix below, determine if the matrix is in row reduced echelon form (RREF) or not. In each case, if the matrix is RREF circle the pivots, and if it is not then explicitly explain which property of RREF is being violated. (3 pts. each)

(a) $\begin{bmatrix}1&1&1\\0&0&0\\0&0&0\end{bmatrix}$ RREF/NOT RREF

(b) $\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}$ RREF/NOT RREF

(c) $\begin{bmatrix}1&2&3\\0&4&5\\0&0&6\end{bmatrix}$ RREF/NOT RREF

(d) $\begin{bmatrix}1&2&0\\0&1&0\\0&0&0\end{bmatrix}$ RREF/NOT RREF

(e) $\begin{bmatrix}1&0&1\\0&1&0\\0&0&0\end{bmatrix}$ RREF/NOT RREF