1. Consider the invertible matrix B below:

$$B = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 1 \\ 4 & 2 & -1 \end{bmatrix}$$

(a) (8 points) Find B^{-1}.

(b) (3 points) Use your B^{-1} to solve $B\vec{x} = (1, 1, 1)$. You *must* use your inverse to solve this to receive points.

(c) (3 points) What is the null space of B? Explain.

(d) (3 points) What is the column space of B, and what is the rank of B? Explain.

(e) (3 points) Is the set of vectors $\beta = \{(1,0,0),\ (0,1,0),\ (0,0,1)\}$ a basis for the column space of B? Explain.

2. (10 points) Consider the matrix:

$$A = \begin{bmatrix} 2 & 1 & 3 & 0 \\ 1 & 3 & -1 & 5 \\ -1 & -1 & -2 & 1 \\ 2 & 0 & 3 & -2 \end{bmatrix}$$

Find the determinant of A. Given your answer, do the columns of A span $\mathbb{R}^4$? Explain.

3. Consider the matrix A and a row equivalent echelon form below:

$$A = \begin{bmatrix} 12 & -3 & 7 & -4 \\ 4 & -1 & 1 & 0 \\ 4 & -1 & 3 & -2 \end{bmatrix} \sim \begin{bmatrix} 4 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) (4 points) Geometrically describe the column space of A, and give a basis for it.

(b) (6 points) Geometrically describe the null space of A, and give a basis for it.

4. Answer the following short questions, being sure to fully explain your answers:

(a) (3 points) If A is an $n \times n$ non-invertible matrix, can its columns serve as a basis for $\mathbb{R}^n$?

(b) (3 points) Is the line $x_2 = 3x_1 - 4$ a subspace of $\mathbb{R}^2$?

(c) (4 points) Let $\vec{b}_1 = (2, -1)$, $\vec{b}_2 = (-2, 2)$, $\vec{a}_1 = (1, -1)$, $\vec{a}_2 = (4, -1)$, $\beta = \{\vec{b}_1, \vec{b}_2\}$, and $\alpha = \{\vec{a}_1, \vec{a}_2\}$. If the coordinates of $\vec{x}$ relative to basis β are $(3, 2)$, what is $\vec{x}$? What are the coordinates of $\vec{x}$ relative to basis α?

(d) (3 points) If A and B are both $n \times n$ matrices, is $(A+B)(A-B) = A^2 - B^2$?

(e) (3 points) If A is a 4×4 matrix with columns $\vec{a}_1$, $\vec{a}_2$, $\vec{a}_3$, and $\vec{a}_4$, and these columns satisfy $\vec{a}_1 + \vec{a}_2 + \vec{a}_3 + \vec{a}_4 = \vec{0}$, can A have a determinant of -7?

(f) (4 points) Let $A = \begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$. Do A and B commute?