1. Show your work for the following problems.

   (a) For the matrix A below, state all of the values of h for which A is invertible. Then, find A^{-1} for $h = 3$.

$$A = \begin{pmatrix} 1 & 1 & -2 \\ 0 & 1 & -1 \\ 2 & 4 & h \end{pmatrix}$$

(b) Let $B = \{\mathbf{v}_1, \mathbf{v}_2\}$ be a basis for a subspace V of $\mathbb{R}^3$, with $\mathbf{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 5 \end{pmatrix}$ and $\mathbf{v}_2 = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$.

Let $\mathbf{u} = \begin{pmatrix} 4 \\ 1 \\ 17 \end{pmatrix}$. What is $[\mathbf{u}]_B$?

(c) Describe the subspace V in part (b) geometrically.

2. (a) Consider the following two matrices:

$$A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}, \qquad B = \begin{pmatrix} 2g & 2h & 2i \\ a+d & b+e & c+f \\ a & b & c \end{pmatrix}.$$

Suppose $\det(B) = 5$. What is $\det(A)$? Give a short explanation.

(b) Let

$$C = \begin{pmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{pmatrix}, \qquad D = \begin{pmatrix} 1 & 1 & 4 & 0 \\ 5 & 0 & -1 & 2 \end{pmatrix}$$

Compute $(CD)^T$ or say that it is not defined.

3. Compute the volume of the parallelepiped in $\mathbb{R}^4$ whose edges are formed by the following vectors:

$$\begin{pmatrix} 3 \\ 3 \\ -6 \\ 6 \end{pmatrix}, \begin{pmatrix} 4 \\ 0 \\ 0 \\ 8 \end{pmatrix}, \begin{pmatrix} -3 \\ 1 \\ -4 \\ -4 \end{pmatrix}, \begin{pmatrix} -1 \\ -3 \\ 3 \\ -1 \end{pmatrix}.$$

4. Consider the following matrix A. It is row equivalent to the given matrix B:

$$A = \begin{pmatrix} 0 & 0 & 2 & 1 & 1 & 1 \\ 1 & 2 & 0 & 2 & 2 & 2 \\ 1 & 2 & 1 & 2 & 0 & 1 \\ 1 & 2 & 3 & 3 & 1 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 & 0 & 0 & -8 & -4 \\ 0 & 0 & 1 & 0 & -2 & -1 \\ 0 & 0 & 0 & 1 & 5 & 3 \\ 0 & 0 & 1 & 0 & -2 & -1 \end{pmatrix}$$

(a) Find a basis for Nul(A).

(b) What is the dimension of Nul(A)?

(c) Find a basis for $\mathrm{Col}(A)$.

(d) What is the rank of A?

5. Let A be a nonzero 4×4 matrix with column vectors $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4$. For each of the parts below, does the statement imply that A is **always invertible**? Justify your answers with a short explanation or counterexample.

   (a) $\mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3 + \mathbf{a}_4 = \mathbf{0}$.

   (b) For all $\mathbf{b}$ in $\mathbb{R}^4$, the augmented matrix $[A \mid \mathbf{b}]$ has 4 pivots.

   (c) $\mathrm{Nul}(A)$ can be described geometrically by a line.

   (d) The dimension of $\mathrm{Col}(A)$ is 2

   (e) The determinant of A is 5.