Math 1553 **Exam 2** Fall 2016

Instructor: Sal Barone (A)

Name: ______________________________

GT username: ________________________

1. No books or notes are allowed.
2. All calculators and/or electronic devices are not allowed.
3. Show all work and fully justify your answer to receive full credit.
4. Please BOX your answers.
5. Good luck!

Page	Max. Possible	Points
1	20	
2	20	
3	26	
4	16	
5	18	
Total	100	

1. Let A be the matrix

$$A = \begin{bmatrix} 4 & 8 & 2 & 0 \\ 1 & 2 & -1 & 2 \\ 0 & 0 & 1 & 4 \\ 2 & 4 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Find Nul(A) the null space of A. Be specific in your answer. (10 pts.)

(b) Find a basis for Col(A) the column space of A. (6 pts.)

(c) Describe Nul(A) geometrically in a few words. (2 pts.)

(d) Describe Col(A) geometrically in a few words. (2 pts.)

2. Let $A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$.

(a) Find A^{-1}. (8 pts.)

(b) Find the coordinate vector $\mathbf{x}$ of $v = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$ in the basis $\left\{ \begin{bmatrix} 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix} \right\}$.

Hint: these are the columns of A. (8 pts.)

(c) Does $\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ belong to the column space of A for any $u_1, u_2 \in \mathbb{R}$? Justify your answer for full credit. (4 pts.)

3. Let A be the matrix

$$A = \begin{bmatrix} 2 & 0 & 2 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}.$$

(a) Find A^{-1}. (12 pts.)

(b) Find $\det(A)$. (10 pts.)

(c) Check your answer to part *(a)* by appealing to the definition of inverse. (4 pts.)

4. Find a basis for (8 pts.)

$$\text{span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix} \right\}.$$

5. If a 6×4 matrix A has exactly 3 pivot positions, then the null space Nul(A) is a subspace of $\mathbb{R}^k$ and the column space Col(A) is a subspace of $\mathbb{R}^\ell$. In this problem you will specify the values of k, ℓ, and state the rank and nullity of A. (2 pts. each)

(a) Nul(A) is a subspace of $\mathbb{R}^k$. Specify the value of k.

(b) Col(A) is a subspace of $\mathbb{R}^\ell$. Specify the value of ℓ.

(c) What is the dimension of Nul(A) the null space of A?

(d) What is the rank of A?

6. Suppose A is a 2×2 matrix and the null space $\text{Nul}(A)$ is the line in $\mathbb{R}^2$ given by the equation $y = 3x$.

(a) What is $\det(A)$? Justify your answer for full credit. (4 pts.)

(b) What is the rank of A? Justify your answer for full credit. (4 pts.)

7. Suppose A and B are square 2×2 matrices and you can assume that A, B, and $A + B$ are all invertible, and that $AB = BA$. Find the matrix equal to the following expression, that is, simplify the following expression. (5 pts.)

$$(A + B)^{-1}[(A^2 - B^2)A^{-1} - (A + B)]A$$

8. Suppose A is any 3×3 matrix such that the ref of A is $A \sim \begin{bmatrix} 1 & 3 & 2 \\ 0 & 5 & 2 \\ 0 & 0 & 4 \end{bmatrix}$. Is it true that a basis for $\text{Col}(A)$ is $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix} \right\}$? Either give a counter-example or justify your answer in some way for full credit. (5 pts.)