Quiz 4 (11 am)

1. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation which associates to each $\mathbf{x} \in \mathbb{R}^2$ the vector obtained from $\mathbf{x}$ by first rotating $\mathbf{x}$ by 90° counter-clockwise and then reflecting the result about the horizontal x-axis. Find the standard matrix A of T as well as the image $T\left(\begin{bmatrix}1\\1\end{bmatrix}\right)$.

Hint: the first column of A is $T\left(\begin{bmatrix}1\\0\end{bmatrix}\right)$ and the second column of A is $T\left(\begin{bmatrix}0\\1\end{bmatrix}\right)$. (4 pts. ea.)

2. Determine whether the given vectors are linearly independent or linearly dependent. If the vectors are linearly dependent find a non-trivial linear combination of the vectors which give the zero vector. (8 pts.)

$$\begin{bmatrix}1\\-1\\0\end{bmatrix}, \begin{bmatrix}2\\2\\1\end{bmatrix}, \begin{bmatrix}6\\2\\2\end{bmatrix}$$

3. True or False section. (1 pt. each)

T/F If A is a 4×3 matrix with 3 pivots, then the columns of A are linearly independent.

T/F If $Ax = 0$ has the trivial solution, then the columns of A are linearly independent.

T/F If the columns of A are linearly independent, then $Ax = b$ has a unique solution.

T/F The linear transformation with standard matrix $\begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix}$ rotates vectors in $\mathbb{R}^2$ by 90° counter-clockwise.