Math 1553 **Exam 1** Spring 2017

Instructor: Sal Barone (A)

Name: _____

GT username: _____

1. No books or notes are allowed.
2. All calculators and/or electronic devices are not allowed.
3. Show all work and fully justify your answer to receive full credit.
4. Please BOX your answers.
5. Good luck!

| Page | Max. Possible | Points |
|---|---|---|
| 1 | 24 | |
| 2 | 20 | |
| 3 | 20 | |
| 4 | 18 | |
| 5 | 18 | |
| Total | 100 | |

**1.** Solve the system of linear equations. (12 pts.)

$$\begin{aligned} -x - y \quad & +z = -1 \\ 3x + y \quad & = 7 \\ -5x - 2y \quad & = -11 \end{aligned}$$

**2.** Is the vector $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ in span $\left\{ \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 9 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \\ 1 \end{bmatrix} \right\}$? Justify your answer for full credit. (12 pts.)

**3.** Find the standard matrix of the linear transformation (10 pts.)

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+2y \\ y+z \\ x+2y \end{bmatrix}$$

**4.** Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation which first rotates a vector in $\mathbb{R}^2$ by $90^\circ$ **clockwise**, then reflects the resulting vector across the line $y = -x$. Find the standard matrix of $T$. (10 pts.)

**5.** Suppose $A$ is row equivalent to

$$A \sim \begin{bmatrix} 1 & 0 & -2 & 8 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ -1 & 0 & 2 & -8 & 1 \end{bmatrix}.$$

Write the solutions of $A\mathbf{x} = 0$ in **parametric vector form**. (10 pts.)

**6.** Give an example of a $3 \times 4$ matrix with 3 rows and 4 columns whose columns span $\mathbb{R}^3$. You must clearly justify that your answer is correct or explain why this is not possible in a few words for full credit. (10 pts.)

**7.** Are the vectors $\begin{bmatrix}1\\0\\1\end{bmatrix}$, $\begin{bmatrix}1\\2\\0\end{bmatrix}$, $\begin{bmatrix}1\\0\\0\end{bmatrix}$ linearly independent or linearly dependent? Fully justify your answer for full credit. (10 pts.)

**8.** Give an example of three vectors $\mathbf{v}$, $\mathbf{w}$, and $\mathbf{b}$ in $\mathbb{R}^4$ such that $\mathbf{b}$ is **not** in span$\{\mathbf{v}, \mathbf{w}\}$ but the set $\{\mathbf{v}, \mathbf{w}, \mathbf{b}\}$ is a linearly dependent set of vectors. Your answer must be clearly justified. (8 pts.)

**9.** For each 3×3 matrix below, determine if the matrix is in row reduced echelon form (RREF) or not. In each case, if the matrix is RREF circle the pivots, and if it is not then explicitly explain which property of RREF is being violated. (3 pts. each)

**(a)** $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ RREF/NOT RREF

**(b)** $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{bmatrix}$ RREF/NOT RREF

**(c)** $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ RREF/NOT RREF

**(d)** $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ RREF/NOT RREF

**(e)** $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ RREF/NOT RREF

**(f)** $\begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ RREF/NOT RREF