Math 1553 **Exam 2** Spring 2017

Instructor: Sal Barone

Name: ________________________________

GT username: __________________________

1. No books or notes are allowed.
2. All calculators and/or electronic devices are not allowed.
3. Show all work and fully justify your answer to receive full credit.
4. Please BOX your answers.
5. Good luck!

Page	Max. Possible	Points
1	20	
2	30	
3	22	
4	16	
5	12	
Total	100	

1. *Two parts.* If $T : \mathbb{R}^5 \to \mathbb{R}^4$ is a linear transformation with standard matrix A, and the dimension of the null space of A is $\dim \operatorname{nul}(A) = 3$, then what is the dimension of the range of T? Justify your answer for full credit. Also, describe the range of T geometrically.

(8 pts.)

2. Find a basis for the null space of the matrix A. (12 pts.)

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 2 & 0 & 0 & 0 & 2 \end{bmatrix}$$

3. Find a basis for W. (8 pts.)

$$W = \text{span}\left\{ \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 6 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ -5 \end{bmatrix} \right\}.$$

4. For the three parts of this problem use the matrix A below.

$$A = \begin{bmatrix} 2 & 3 & 0 \\ -4 & 6 & 1 \\ 10 & 3 & 7 \end{bmatrix}$$

(a) Find the LU decomposition of A. (10 pts.)

(b) Find the determinant of A. Show your work. *Hint: use part (a)* (8 pts.)

(c) Find all solutions to $A\mathbf{x} = 0$. Justify your answer for full credit. (4 pts.)

5. *True or False.* If v_1, v_2, v_3 are vectors in $\mathbb{R}^3$ such that none are scalar multiples of each other, then the set $\{v_1, v_2, v_3\}$ is a basis for $\mathbb{R}^3$. Either give a counterexample and explain why the statement is false, or give a clear justification for why the statement is true.

(8 pts.)

6. Find the inverse of A. Check your answer by matrix multiplication for full credit.

(14 pts.)

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

7. Find the standard coordinates of the vector $\begin{bmatrix} 3 \\ -2 \end{bmatrix}_{\mathcal{B}}$ where $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix} \right\}$. (8 pts.)

8. Suppose the determinant of the matrix $\begin{vmatrix} a & b & c \\ 4 & 3 & 2 \\ 1 & 5 & 6 \end{vmatrix} = 5$. What is the determinant of $\begin{bmatrix} 4 & 3 & 1 \\ 1 & 5 & 6 \\ 3a & 3b & 3c \end{bmatrix}$? (8 pts.)

9. Suppose A is a 3×3 matrix and $\det(A^2) = 1$. Justify your answer to the following questions for full credit. (3 pts. each)

(a) *True or False.* A is invertible.

(b) *True or False.* The determinant of A is $\det(A) = 1$.

(c) *True or False.* The columns of A span $\mathbb{R}^3$.

(d) *True or False.* There are infinitely many solutions to $A\mathbf{x} = \mathbf{b}$ for some choice of $\mathbf{b}$ in $\mathbb{R}^3$.