Math 1553 **Exam 2** Spring 2017

Instructor: Sal Barone

Name: ______________________________

GT username: ________________________

1. No books or notes are allowed.
2. All calculators and/or electronic devices are not allowed.
3. Show all work and fully justify your answer to receive full credit.
4. Please BOX your answers.
5. Good luck!

Page	Max. Possible	Points
1	15	
2	25	
3	20	
4	24	
5	16	
Total	100	

1. Diagonalize the matrix. (15 pts.)

$$A = \begin{bmatrix} 2 & 0 & -3 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

2. Find all eigenvalues of the matrix. (15 pts.)

$$A = \begin{bmatrix} 6 & -4 & 2 \\ 2 & 2 & 0 \\ 4 & -4 & 4 \end{bmatrix}$$

3. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation which first rotates vectors in $\mathbb{R}^2$ by $180\,^\circ$, and then projects the result to the x-axis. Find two linearly independent eigenvectors of the standard matrix of T and state the associated eigenvalues. (10 pts.)

4. Find all complex eigenvalues of the matrix A, and find an associated eigenvector for each eigenvalue. (10 pts.)

$$A = \begin{bmatrix} 3 & -2 \\ 2 & 3 \end{bmatrix}$$

5. Let A be a 2×2 matrix which satisfies $A\mathbf{v}_1 = 2\mathbf{v}_1$ and $A\mathbf{v}_2 = \mathbf{v}_2$ where $\mathbf{v}_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. If $\mathbf{x}$ is the vector $\mathbf{x} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$, compute $A^4\mathbf{x}$. (10 pts.)

6. The matrix $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ has eigenvalues $\lambda = a + b$ and $\lambda = a - b$. Find an associated eigenvector for each eigenvalue. You may assume that neither a nor b is zero. (12 pts.)

7. Suppose A is a 2×2 matrix with $\lambda = 0$ and $\lambda = 1$ as eigenvalues. Show that $A^2 = A$. (12 pts.)

8. If the eigenvalues of A are -1, 3, and 5, then what are the eigenvalues of A^3? (8 pts.)

9. Show that the statement is true or give a counterexample: If A has n linearly independent eigenvectors, then A has n distinct eigenvalues. (8 pts.)