Today

* go over syllabus/myLab/ daily routine
* math stuff
  - ↳ matrices, augmented matrix
  - ↳ row operations
  - ↳ # of solutions: infinitely many, none, or unique.

FRIDAY in recitation
Quiz every week (unless an exam)
Turn in Handwritten HW (on website)

Systems of linear equations.

Ex. Solve. $\begin{cases} x - y = 1 \\ 2x + y = 8 \end{cases}$

To solve I mean we have to find the x and y values that make both equations true at the same time.

Substitution method.

Solve for one of the variables & substitute back in

$x = 1 + y$ $\quad x = 3.$

$2(1+y) + y = 8$

$2 + 2y + y = 8$

$3y = 6 \quad y = 2$

Soln

$x = 3$

$y = 2$

Elimination method

multiply each equation by some number so that one the variables "cancels" when you add them

$$\begin{array}{rl} & x - y = 1 \\ + & 2x + y = 8 \\ \hline & 3x = 9 \end{array}$$

so $\underline{\underline{x = 3}}$

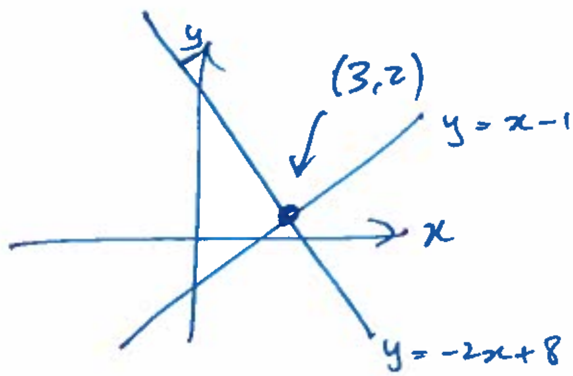
$$3 - y = 1$$

so $\underline{\underline{y = 2}}$

Graphing

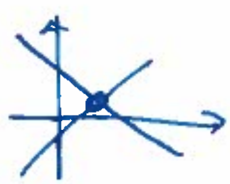
$y = x - 1$

$y = -2x + 8$



Q: In what ways can two lines intersect?

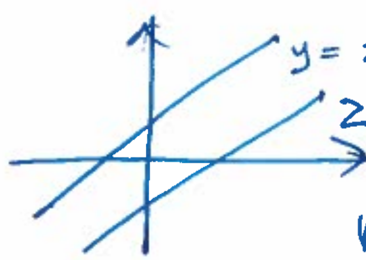
1) at a point



<u>unique intersection point</u>

<u>Unique</u> soln

2) not at all



<u>no soln</u>

3) infinitely many times

Matrices are the **BEST** way to solve any except the simplest systems. For example the previous system can be represented as

$$[A \mid b] = \left[\begin{array}{cc|c} 1 & -1 & 1 \\ 2 & +1 & 8 \end{array}\right]. \qquad \begin{cases} x - y = 1 \\ 2x + y = 8 \end{cases}$$

↓ row reduce

$$\sim \begin{array}{r} \\ R_1+R_2 \rightarrow \end{array} \left[\begin{array}{cc|c} 1 & -1 & 1 \\ 3 & 0 & 9 \end{array}\right] \qquad \begin{cases} x - y = 1 \\ 3x \quad = 9 \end{cases}$$

row reduce ↓

$$\sim \begin{array}{r} \\ \tfrac{1}{3}R_2 \end{array} \left[\begin{array}{cc|c} 1 & -1 & 1 \\ 1 & 0 & 3 \end{array}\right] \qquad \begin{cases} x - y = 1 \\ x \quad = 3 \end{cases}$$

$$\sim \begin{array}{r} -R_2+R_1 \rightarrow \\ \\ \end{array} \left[\begin{array}{cc|c} 0 & -1 & -2 \\ 1 & 0 & 3 \end{array}\right] \qquad \begin{cases} -y = -2 \\ x = 3 \end{cases}$$

$$\sim \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & -1 & -2 \end{array}\right] \qquad \begin{cases} x = 3 \\ -y = -2 \end{cases}$$

$$\sim \begin{array}{r} \\ -R_2 \end{array} \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 2 \end{array}\right].$$

The three types of (allowable) row operations

1) switch two rows $R_i \leftrightarrow R_j$

2) mult. row by nonzero #
$c \cdot R_i \rightarrow R_i$

3) add a multiple of one row to another row
$cR_i + R_j \rightarrow R_j$

↑ doing these don't change the solutions to a system of linear equations.

1)
$$\left[\begin{array}{cc|c} 0 & 2 & 3 \\ 1 & 4 & 5 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 4 & 5 \\ 0 & 2 & 3 \end{array}\right]$$

$0x + 2y = 3$
$x + 4y = 5$

$x + 4y = 5$
$0x + 2y = 3$

2)
$$\left[\begin{array}{cc|c} 3 & 6 & 10 \\ 0 & 1 & 4 \end{array}\right] \sim_{\frac{1}{3}R_1} \left[\begin{array}{cc|c} 1 & 2 & 10/3 \\ 0 & 1 & 4 \end{array}\right]$$

$3x + 6y = 10$
$y = 4$

$x + 2y = 10/3$
$y = 4$

How do you know when are DONE row reducing. "RREF reduced row echelon form"

AKA

What is the ideal form of a matrix which represents a system of lin eqns?

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 2 & 1 & 7 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right] \sim \xrightarrow{-2R_1+R_2} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

$x \quad + 2z = 1$
$2x + y + 7z = 4$
$0 = 0$

$$\underbrace{\begin{array}{l} x + 2z = 1 \\ y + 3z = 2 \\ 0 = 0 \end{array}}_{\text{simpler so better for solving}}$$

Infinitely many solutions?

Solve.

Ex. $3x + y = 10$

$9x + 3y = 30$

$$\left[\begin{array}{cc|c} 3 & 1 & 10 \\ 9 & 3 & 30 \end{array}\right] \sim \xrightarrow{-3R_1 + R_2} \left[\begin{array}{cc|c} 3 & 1 & 10 \\ 0 & 0 & 0 \end{array}\right].$$

$3x + y = 10$
$9x + 3y = 30$

$3x + y = 10$
$0 = 0$.

$y =$ free

$x = \dfrac{10 - y}{3}$

$$\sim \xrightarrow{\frac{1}{3}R_1} \begin{array}{ccc} \text{pivot col.} & \text{free col.} & \\ \left[\begin{array}{cc|c} 1 & 1/3 & 10/3 \\ 0 & 0 & 0 \end{array}\right] & & \end{array}.$$

$x + \frac{1}{3}y = \frac{10}{3}$

$$\boxed{\begin{aligned} x &= \frac{10}{3} - \frac{1}{3}y \\ y &= \text{free} \end{aligned}}$$

Defn. A matrix is REF (row echelon form) if

1) all zero rows at the bottom
2) ~~all~~ the leading entries in each row form a staircase (so are below and to the right of those above)
3) Below each leading entry is 0.

For RREF (reduced row ~~ech~~echelon form)

4) leading entries are all 1.

Examples: Which are REF/RREF?

$$\begin{bmatrix} 2 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

REF / RREF / NOT

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

REF / RREF / NOT

$$\begin{bmatrix} 0 & 0 & 0 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{bmatrix}$$

REF / RREF / NOT

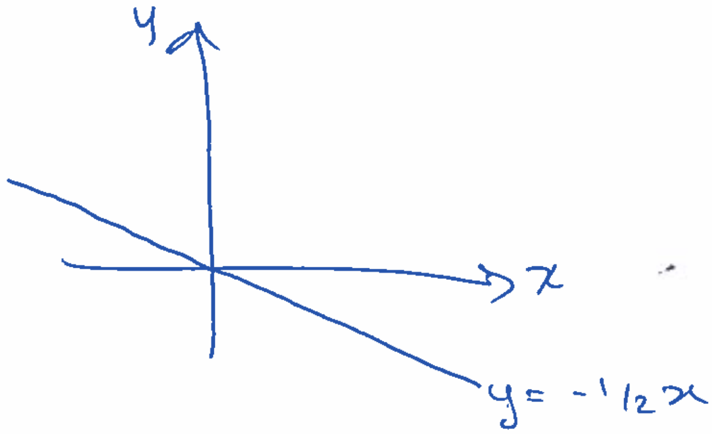
$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The RREF of any matrix is UNIQUE.

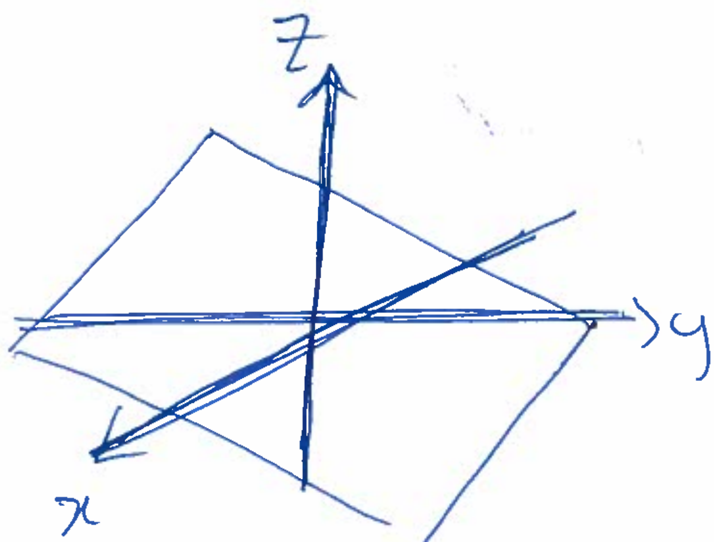
What is it? $x + 2y + 3z = 0$

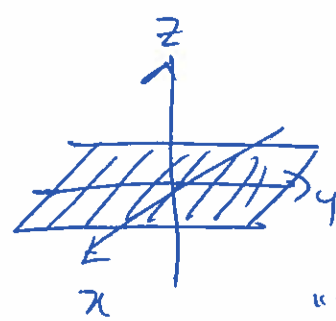
What is it? $x + 2y = 0$ a line.

$x + 2y = 0$ describes geometrically a "line in $\mathbb{R}^2$"

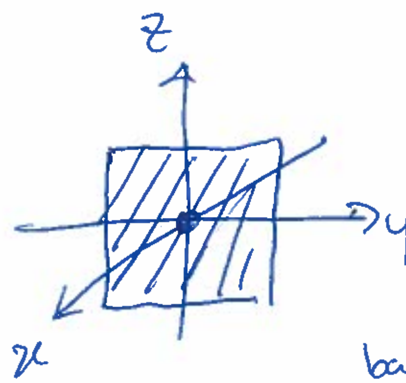


$x + 2y + 3z = 0$ describes a "plane in $\mathbb{R}^3$"

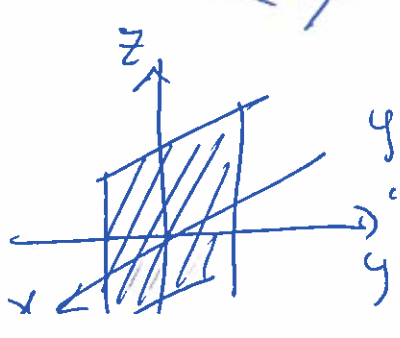




$z = 0$ "The floor"



$x = 0$ the back wall or "chalkboard"



$y = 0$ "sidewall"

Follow-up question:

How can 2 planes in $\mathbb{R}^3$ intersect?

1) not at all (parallel but not touching)
(giving no soln to two eqns in 3 unknowns)

2) in a plane (same plane twice)

3) in a line.

$$\begin{array}{cccc} x & y & z & \text{constant} \\ \end{array}$$
$$\left[\begin{array}{ccc|c} \circ & \circ & \circ & \circ \\ \circ & \circ & \circ & \circ \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & * & * \\ 0 & 1 & * & * \end{array}\right]$$

If # eqns < # unknowns

$\Longrightarrow$ no unique soln.
(∞-many or none both possible).