Wednesday 1/18

Row reduction algorithm (on slide)

~~Step 1a swap 1st row with a lower one so that leftmost non-zero entry is 1~~

Solve

Ex. $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 9 \\ 2 & -1 & 1 & 8 \\ 3 & 0 & -1 & 3 \end{array}\right] \sim \begin{array}{r} \\ -2R_1+R_2 \\ -3R_1+R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 9 \\ 0 & -5 & -5 & -10 \\ 0 & -6 & -10 & -24 \end{array}\right]$

$\sim \frac{-1}{5}R_2 \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 9 \\ 0 & 1 & 1 & 2 \\ 0 & -6 & -10 & -24 \end{array}\right] \sim 6R_2+R_3 \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 9 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & -4 & -12 \end{array}\right]$

$\sim -\frac{1}{4}R_3 \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 9 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$

Start eqns.

$$\boxed{\begin{aligned} x + 2y + 3z &= 9 \\ 2x - y + z &= 8 \\ 3x \quad - z &= 3 \end{aligned}}$$

$\sim -2R_2+R_1 \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 5 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$

ref eqns.

$$\begin{aligned} x + 2y + 3z &= 9 \\ y + z &= 2 \\ z &= 3 \end{aligned}$$

$\sim \begin{array}{r} -R_3+R_1 \rightarrow \\ -R_3+R_2 \rightarrow \\ \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$

$\boxed{\bigstar = \beta} \quad \# = \emptyset$

So $\boxed{x = 2,\ y = -1,\ z = 3}$ is the unique soln.

Then $\boxed{\bigstar + \# = \beta + \emptyset}$

<u>Ex</u> only two planes in $\mathbb{R}^3$

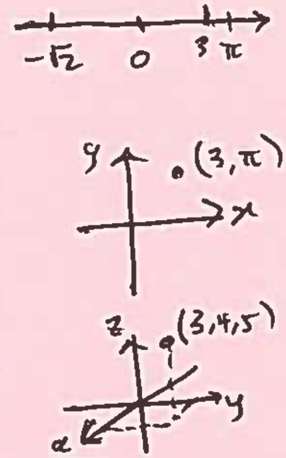
<u>Solve</u>.

$$\begin{cases} x - y + 3z = -1 \\ 0x + y + 2z = 1 \end{cases}$$

$\mathbb{R}$

$\mathbb{R}^2$

$\mathbb{R}^3$



$$\left[\begin{array}{ccc|c} 1 & -1 & 3 & -1 \\ 0 & 1 & 2 & 1 \end{array}\right]$$

$x + y = 1$

$$\sim \; R_2 + R_1 \rightarrow \begin{array}{c} \begin{array}{cccc} & & s\downarrow & \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 5 & 0 \\ 0 & 1 & 2 & 1 \end{array}\right] \end{array} .$$

$$x + 5z = 0$$
$$y + 2z = 1$$

$x + 5s = 0$

$y + 2s = 1$

$z = s$ (free)

Suppose I want 3 solns

✓ $z = 0 \quad x = 0 \quad y = 1$

✓ $z = -2 \quad x = 10 \quad y = 5$

✓ $z = 3 \quad x = -15 \quad y = -5$

$$\begin{cases} x = -5s \\ y = 1 - 2s \\ z = s \text{ (free)} \end{cases}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -5s \\ 1-2s \\ s \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} -5 \\ -2 \\ 1 \end{bmatrix}$$

You do it

Find parametric eqn form for the solns to

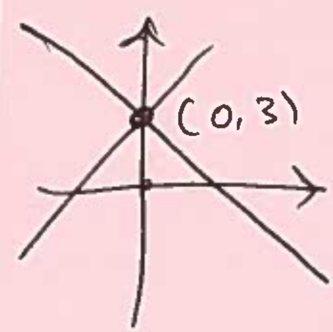
(Similar to what we just did
$x =$ ___
$y =$ ___
$z = s$ (free))

$$\left[\begin{array}{cccc|c} 1 & 0 & 5 & 2 & 0 \\ 2 & 0 & 10 & 1 & 0 \end{array}\right]$$

$$\sim_{-2R_1+R_2} \left[\begin{array}{cccc|c} 1 & 0 & 5 & 2 & 0 \\ 0 & 0 & 0 & -3 & 0 \end{array}\right] \sim_{-\frac{1}{3}R_2} \left[\begin{array}{cccc|c} 1 & 0 & 5 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$\sim_{-2R_2+R_1} \begin{array}{c} \begin{array}{cccc} \text{pivot} & r\ \text{free} & s\ \text{free} & \text{pivot} \end{array} \\ \left[\begin{array}{cccc|c} 1 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \end{array}.$$

Q: I'm worried that 4th variable is zero.



Write solns in terms of free variables, solving for pivot variables.

$$x + 5s = 0$$
$$y = r \quad \text{(free)}$$
$$z = s \quad \text{(free)}$$
$$w = 0$$

$$x = -5s$$
$$\left.\begin{array}{l} y = r \\ z = s \end{array}\right\} \text{free}$$
$$w = 0$$

What about

$$\begin{bmatrix} 1 & 0 & 5 & 2 \\ 2 & 0 & 10 & 1 \end{bmatrix}$$

<u>means</u>

$$\left[\begin{array}{cccc|c} 1 & 0 & 5 & 2 & 0 \\ 2 & 0 & 10 & 1 & 0 \end{array}\right].$$

<u>NOT</u>

$$\left[\begin{array}{ccc|c} 1 & 0 & 5 & 2 \\ 2 & 0 & 10 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 5 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$$x + 5z = 0$$
$$0 = 1$$

no solution to a system like this

$$\left[\begin{array}{ccc|c} * & * & * & * \\ 0 & 0 & 0 & \neq 0 \end{array}\right]$$

<u>inconsistent</u> means no solution possible.