# Worksheet 2, Math 1553

Sections from Lay 5th edition: 1.3, 1.4

1. Consider the following vectors:

$$\vec{u}_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \ \vec{u}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \ \vec{u}_3 = \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix}, \ \vec{b} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}.$$

   Is $\vec{b}$ in the span of $\{\vec{u}_1,\ \vec{u}_2,\ \vec{u}_3\}$? Is $\vec{b}$ a linear combination of vectors $\vec{u}_1$, $\vec{u}_2$, and $\vec{u}_3$? Is the linear system with augmented matrix
   $A = \begin{bmatrix} \vec{u}_1 & \vec{u}_2 & \vec{u}_3 & \vec{b} \end{bmatrix}$ consistent?

2. Write vector $\begin{bmatrix} 6 \\ 11 \\ 6 \end{bmatrix}$ as a linear combination of the vectors $\vec{u} = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}$.

3. Mark each statement as true or false, and justify your answers:

   (a) A vector $\vec{b}$ is a linear combination of the columns of a matrix $A$ if and only if the equation $A\vec{x} = \vec{b}$ has at least one solution.

   (b) The equation $A\vec{x} = \vec{b}$ is consistent if the augmented matrix $\begin{bmatrix} A & \vec{b} \end{bmatrix}$ has a pivot position in every row.

   (c) The first entry in the product $A\vec{x}$ is a sum of products.

   (d) If the columns of an $m \times n$ matrix $A$ span $\mathbb{R}^m$, then the equation $A\vec{x} = \vec{b}$ is consistent for each $\vec{b}$ in $\mathbb{R}^m$.

   (d) If $A$ is an $m \times n$ matrix and if the equation $A\vec{x} = \vec{b}$ is inconsistent for some $\vec{b}$ in $\mathbb{R}^m$, then $A$ cannot have a pivot position in every row.

4. Describe the span of the vectors. If the span is a line or a plane, find the equation.

   (a) $\vec{v}_1 = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 1 \\ -1 \\ -4 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 1 \\ 0 \\ -5 \end{bmatrix}.$

   (b) $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} -3 \\ -6 \\ -9 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix}.$