Worksheet 5, Math 1553

Sections from Lay 5^{th} edition: 2.1, 2.2

Exercises

1. Consider the matrices
$$A = \begin{pmatrix} 2 & 5 \\ -3 & 1 \end{pmatrix}, \ B = \begin{pmatrix} 4 & -5 \\ 3 & k \end{pmatrix}.$$
For what value(s) of k, if any, do matrices A and B commute?

2. Suppose the last column of the product AB is a column of zeros, but matrix B does not have a column of zeros. What can we say about the columns of matrix A?

3. If possible, compute the inverse of the matrix. For what values of p does the inverse exist?
$$\begin{pmatrix} 1 & 0 & -1 \\ -3 & 1 & 3 \\ 2 & -3 & p \end{pmatrix}$$

4. True or false. Justify your reasoning. If the statement is false, identify a counterexample.
   (a) The transpose of any sum of matrices is always equal to the sum of their transposes.
   (b) The transpose of any product of matrices is always equal to the product of their transposes.
   (c) If A is a square matrix, then $\left(A^2\right)^T = \left(A^T\right)^2$.
   (d) If A and B are matrices, and the product AB is equal to the zero matrix, then A and/or B must also be a zero matrix.

5. Consider A a 3×3 matrix, and $I = I_3$ the 3×3 identity matrix.
   (a) Denote row i of the 3×3 identity matrix as $\mathrm{row}_i(I)$. What is $\mathrm{row}_i(I)A$, for $i = 1, 2, 3$, equal to?
   (b) If rows 1 and 2 of A are interchanged, the result can be expressed as EA, where E is an elementary matrix obtained by interchanging the rows 1 and 2 of I. What is E?
   (c) If row 3 of A is multiplied by 5, state why the result can be expressed as EA, where E is formed by multiplying row 3 of I by 5.
   (d) If row 3 of A is replaced by $\mathrm{row}_3(A) - 4\mathrm{row}_1(A)$,state why the result is EA, where E is formed from I by replacing row 3 of I by $\mathrm{row}_3(I) - 4\mathrm{row}_1(I)$.