Worksheet 6, Math 1553

Sections from Lay 5^{th} edition: 2.3, 2.5

Exercises

1. Answer the following short questions, justifying your answers. Note that all referenced matrices are $n \times n$ square.
   (a) Matrix A has one column that is 7 times another column. Is A invertible?
   (b) A is the standard matrix of a linear transform $T: \mathbb{R}^n \to \mathbb{R}^n$ that is one-to-one. Is A invertible?
   (c) A is not invertible. How many solutions are there to the equation $A\vec{x} = \vec{0}$?
   (d) There are some vectors in $\mathbb{R}^n$ that are not in the span of the columns of A. Is A invertible?
2. If possible, give an example of the following.
   (a) A matrix that has an LU factorization where L is 3×3 and U is 3×2.
   (b) A 3×3 matrix that is singular and has exactly two pivot columns.
   (c) A matrix that is singular and has an LU decomposition.
   (d) A matrix that is invertible and does not have an LU decomposition.
   (e) A matrix that is singular and does not have an LU decomposition.
3. $A = \begin{pmatrix} 1 & -1 & 2 \\ 3 & -5 & 5 \end{pmatrix}$
   (a) Compute the LU decomposition of A.
   (b) Use your LU decomposition to solve $A\vec{x} = \vec{b}$, where $\vec{b} = \begin{pmatrix} 2 \\ 13 \end{pmatrix}$
4. If possible, fill in the missing elements of the matrices below with numbers so that each of the matrices are invertible. If it is not possible to do so, state why.

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & & \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ & & 1 \end{pmatrix}, \qquad C = \begin{pmatrix} 1 & 1 & \\ 1 & 1 & \\ 1 & 1 & 1 \end{pmatrix}$$