Worksheet 7, Math 1553

Sections from Lay 5^{th} edition: 2.8, 2.9, 3.1 and 3.2.

Exercises

1. Consider the matrix
$$A = \begin{pmatrix} 0 & -1 & 2 \\ 1 & 5 & -9 \\ 4 & 1 & 2 \end{pmatrix}.$$
   (a) Construct bases for the column space of A and for the null space of A.
   (b) What are the dimensions of Col$A$ and Null$A$?
   (c) Describe Col$A$ and Null$A$ geometrically.
   (d) What is the rank of A?

2. Indicate whether the statement true or false. If it is true, in one or two sentences, explain why. If false, give a counter example or explain why in one or two sentences.
   (a) If M is a 3×5 matrix, and its columns span $\mathbb{R}^3$, then the null space is $\mathbb{R}^2$.
   (b) It is not possible for the null space of an $m \times n$ matrix to be $\mathbb{R}^n$.

3. A 7×5 matrix A has rank 3. What is the dimension of the set of solutions to $A\vec{x} = \vec{0}$?

4. If possible, construct a 3×4 matrix A, in reduced echelon form, whose Column space and Null space both have dimension 2.

5. Consider the vectors
$$\vec{b}_1 = \begin{pmatrix} 1 \\ 5 \\ -3 \end{pmatrix}, \; \vec{b}_2 = \begin{pmatrix} -3 \\ -7 \\ 5 \end{pmatrix}, \; \vec{x} = \begin{pmatrix} 4 \\ 10 \\ -7 \end{pmatrix}.$$
   (a) Explain why the set $\beta = \{\vec{b}_1, \; \vec{b}_2\}$ can be considered as a basis for a subspace H.
   (b) Geometrically describe H.
   (c) Is $\vec{x}$ in H? If so, give the coordinates of $\vec{x}$ relative to the basis β.

6. Compute the determinant by cofactor expansions:
$$\begin{vmatrix} 1 & -2 & 5 & 2 \\ 0 & 0 & 3 & 0 \\ 2 & -4 & -3 & 5 \\ 2 & 0 & 3 & 5 \end{vmatrix}$$

7. Calculate the following determinants and compare to describe how row operations affect the determinant:
$$\begin{vmatrix} a & b \\ c & d \end{vmatrix}, \quad \begin{vmatrix} c & d \\ a & b \end{vmatrix}, \quad \begin{vmatrix} a+kc & b+kd \\ c & d \end{vmatrix}, \quad \begin{vmatrix} a & b \\ kc & kd \end{vmatrix}.$$