Worksheet 8, Math 1553

Sections from Lay 5^{th} edition: 5.1 and 5.2

Exercises

1. (a) Determine whether $\vec{u}$ and $\vec{v}$ are eigenvectors of A. If so, what are their eigenvalues? Do not construct the characteristic polynomial of A.

$$A = \begin{pmatrix} -3 & -3 & 2 \\ 6 & 4 & 0 \\ 5 & 3 & 0 \end{pmatrix}, \quad \vec{u} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}, \quad \vec{v} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

   (b) Is $\lambda = 2$ an eigenvalue of B? Do not compute the characteristic polynomial.

$$B = \begin{pmatrix} 3 & 2 \\ 3 & 8 \end{pmatrix}$$

2. Construct a basis for the eigenspace of

$$A = \begin{pmatrix} 4 & -2 \\ -3 & 9 \end{pmatrix}$$

   with eigenvalue 10.

3. Let $\vec{u}$ and $\vec{v}$ both be eigenvectors of 2×2 matrix A with real eigenvalues λ and μ, respectively, and $\lambda \neq \mu$.

   (a) Explain why the set of vectors $e = \{\vec{u}, \vec{v}\}$ can serve as a basis for $\mathbb{R}^2$.

   (b) If the coordinates of a vector $\vec{x}$ in $\mathbb{R}^2$ relative to the basis e are $(c_1,\ c_2)$, what are the coordinates of the vector $A\vec{x}$ relative to basis e?

   (c) If $\lambda = 0$, what is the rank of A?

4. Consider the matrix

$$A = \begin{pmatrix} 3 & 4 & 5 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}.$$

   Construct the characteristic equation for the eigenvalues of A, and then solve this equation, giving the eigenvalues and their multiplicities.