Sample Midterm 2A, Math 1554

PLEASE PRINT YOUR NAME CLEARLY IN ALL CAPITAL LETTERS

First Name ____________________ Last Name ____________________

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Section Number (e.g. A4, QH3, etc.) _______ TA Name _________________________

Student Instructions

- **Show your work** and justify your answers for all questions unless stated otherwise.
- **Organize your work** in a reasonably neat and coherent way.
- Calculators, notes, cell phones, books are not allowed.
- Use dark and clear writing: your exam will be scanned into a digital system.
- Exam pages are double sided. Be sure to complete both sides.
- Leave a 1 inch border around the edges of exams.
- Any work done on scratch paper will not be collected and will not be graded.

Math 1554, Sample Midterm 2A. Your initials: ______________

You do not need to justify your reasoning for questions on this page.

1. (10 points) Indicate **true** if the statement is true, otherwise, indicate **false**.

	true	false
a) If a square matrix is not invertible, then it does not have an LU factorization.	○	○
b) $H = \{\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^2 \mid x_1 + x_2 = 1\}$ is a subspace.	○	○
c) If $A\vec{x} = A\vec{y}$ for some $\vec{x} \neq \vec{y} \in \mathbb{R}^n$, then A cannot be invertible.	○	○
d) If A is non-singular, then A^n must be non-singular, for any integer $n > 1$.	○	○
e) If A and B are square matrices and $AB = I$, then A is invertible.	○	○
f) All elementary matrices are triangluar.	○	○
g) If $T_A : \mathbb{R}^3 \mapsto \mathbb{R}^4$ is one-to-one, then $\text{rank}(A) = 3$ and $\dim(\text{Null}(A))$ is 0.	○	○
h) A 5×5 matrix A with rank 3 has an eigenvalue $\lambda = 0$.	○	○
i) The algebraic multiplicity of an eigenvalue λ can be zero.	○	○
j) If A is square and row equivalent to an identity matrix, then $\det(A) \neq 0$.	○	○

2. (2 points) Fill in the blanks.

   (a) $T_A = A\vec{x}$, where $A \in \mathbb{R}^{2\times 2}$, is a linear transform that rotates vectors in $\mathbb{R}^2$ clockwise by θ radians about the origin, then reflects them through the line $x_1 = 0$, then projects them onto the x_1-axis. Compute $\det(A)$. ☐

   (b) An eigenvector of $A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$ is $\vec{v}_1 = \begin{pmatrix} 0 \\ -1 \\ 1 \\ 0 \end{pmatrix}$. What is the eigenvalue associated with $\vec{v}_1$? ☐ *Hint: do not compute the characteristic polynomial!*

You do not need to justify your reasoning for questions on this page.

3. (10 points) If possible, write down an example of a matrix with the following properties. If it is not possible to do so, write *not possible*. You do not need to justify your reasoning.

   (a) If possible, give an example of a 2×2 matrix whose column space is the line $x_1 = -2x_2$, and whose null space is the line $x_1 = 4x_2$.

   (b) A 3×7 matrix, A, in RREF, such that $\dim(\text{Col}(A)) = 4$, and $\dim(\text{Null}(A)) = 3$.

   (c) A 3×5 matrix, A, in RREF, such that $\dim(\text{Col}(A)) = 3$.

   (d) A 4×4 lower triangular matrix A, such that $\det(A) = -1$ and $\text{rank}(A) = 4$.

   (e) A 3×3 matrix whose determinant is equal to zero, and whose null space is the plane $x_1 + 2x_2 + 3x_3 = 0$.

4. (3 points) Let A and B be $n \times n$ matrices such that $A^2 = I_n$, $BA = I_n$, and

$$C = \begin{bmatrix} A & B \\ B & -A \end{bmatrix}$$

Express C^2 in terms of A and B. Simplify as much as possible. Show your work.

5. (5 points) If a square matrix A has eigenvalue 2 with eigenvector $\vec{x}$ and eigenvalue $-1/2$ with eigenvector $\vec{y}$, express the the following in terms of $\vec{x}$ and $\vec{y}$. It is not necessary to show your work here.

   (a) $A^3\vec{x} =$ ☐

   (b) $A^3\vec{y} =$ ☐

   (c) $A(\vec{x} + \vec{y}) =$ ☐

   (d) $A^2(\vec{x} + \vec{y}) =$ ☐

   (e) $A^{-1}(\vec{x} + \vec{y}) =$ ☐

6. (3 points) The vector $\vec{x} = (1, 12, 3)^T$ is in the span of $\mathcal{B} = \{\vec{v}_1, \vec{v}_2\}$, where $\vec{v}_1 = (1, 0, -1)^T$ and $\vec{v}_2 = (-1, 3, 2)^T$. Compute $[\vec{x}]_{\mathcal{B}}$.

7. (3 points) Compute the inverse of the matrix $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ -3 & 0 & 3 \end{pmatrix}$.

8. (4 points) Suppose the determinant $\begin{vmatrix} a & b & c \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix} = 5$, where a, b and c are real numbers. What is the determinant below equal to?

$$\begin{vmatrix} 1 & 2 & 3 \\ 2a & 2b & 2c \\ 5 & 7 & 9 \end{vmatrix}$$

9. (4 points) Consider the sequence of row operations that reduce matrix A to echelon form, U.

$$A = \underbrace{\begin{pmatrix} 1 & 2 & 2 \\ -2 & 2 & 0 \\ 1 & 14 & 3 \end{pmatrix}}_{A} \sim \underbrace{\begin{pmatrix} 1 & 2 & 2 \\ 0 & 6 & 4 \\ 1 & 14 & 3 \end{pmatrix}}_{E_1 A} \sim \underbrace{\begin{pmatrix} 1 & 2 & 2 \\ 0 & 6 & 4 \\ 0 & 12 & 1 \end{pmatrix}}_{E_2 E_1 A} \sim \underbrace{\begin{pmatrix} 1 & 2 & 2 \\ 0 & 6 & 4 \\ 0 & 0 & -7 \end{pmatrix}}_{E_3 E_2 E_1 A} = U$$

(a) Construct the elementary matrices E_1, E_2, E_3.

$E_1 =$ $\qquad$ $E_2 =$ $\qquad$ $E_3 =$

(b) Consider the matrix products listed below. Which (if any) represents A?

I) $E_3 E_2 E_1 U$
II) $E_1 E_2 E_3 U$
III) $E_1^{-1} E_2^{-1} E_3^{-1} U$
IV) $E_3^{-1} E_2^{-1} E_1^{-1} U$

10. (4 points) Construct a basis for the eigenspace of A associated with the eigenvalue $\lambda = 3$.

$$A = \begin{pmatrix} 5 & -1 & 2 \\ 2 & 2 & 2 \\ 2 & -1 & 5 \end{pmatrix}$$