MATLAB Exploration #2/#3 for MATH 1554

For each MATLAB assignment, follow the step-by-step formatting guidelines we provided. You will be graded on completeness, following directions, proper usage of comments, and overall readability of your code and published .pdf submission. We recommend `format bank`

For Week 6: MATLAB #2 - Problem 13 and Problem 14 from Lay, Section 2.6, page 139.

The consumption matrix below is based on input-output data for the U.S. economy in 1958, with data for 81 sectors grouped into 7 larger sectors: (1) nonmetal household and personal products, (2) final metal products (such as motor vehicles), (3) basic metal products and mining, (4) basic nonmetal products and agriculture, (5) energy, (6) services, and (7) entertainment and miscellaneous products[1]. (Units are in millions of dollars.)

$$C = \begin{bmatrix} .1588 & .0064 & .0025 & .0304 & .0014 & .0083 & .1594 \\ .0057 & .2645 & .0436 & .0099 & .0083 & .0201 & .3413 \\ .0264 & .1506 & .3557 & .0139 & .0142 & .0070 & .0236 \\ .3299 & .0565 & .0495 & .3636 & .0204 & .0483 & .0649 \\ .0089 & .0081 & .0333 & .0295 & .3412 & .0237 & .0020 \\ .1190 & .0901 & .0996 & .1260 & .1722 & .2368 & .3369 \\ .0063 & .0126 & .0196 & .0098 & .0064 & .0132 & .0012 \end{bmatrix}, \qquad \mathrm{d} = \begin{bmatrix} 74,000 \\ 56,000 \\ 10,500 \\ 25,000 \\ 17,500 \\ 196,000 \\ 5,000 \end{bmatrix}.$$

(a) Check that the matrix C satisfies the conditions of being a consumption matrix by multiplying on the left by the row vector $\mathrm{v} = [1\ 1\ 1\ 1\ 1\ 1\ 1]$. In the comments, explain why the entries of $\mathrm{v} \cdot C$ verify that C is a consumption matrix.

(b) Set up and augmented matrix from the system $\mathrm{x} = C\mathrm{x} + \mathrm{d}$ and row reduce in matlab to find the production levels x needed to satisfy the final demand d.
Hint: to form an augmented matrix $[A \mid \mathrm{b}\,]$ *use command* `M=[A  b]` *in matlab, in particular matlab will let you enter* ***matrices*** *as the entries of a matrix!*

(c) Find the inverse $(I - C)^{-1}$ and use it to find x. Make sure that your answer agrees with part *(b)*[2]. *Hint: use command* `inv(A)` *to get inverses in matlab.*

(d) Compute $L = I + C + C^2 + \ldots + C^k$ for sufficiently large k such that L approximates $(I - C)^{-1}$ sufficiently to ensure that $L \cdot \mathrm{d}$ gives the correct approximation of x up to four significant digits. In the comments, specify the smallest value of k that works.

(e) The demand vector above is reasonable for 1958 data, but Leontief's discussion of the economy in the reference cited here used a demand vector closer to 1964 data: d_2=[99640 ; 75548 ; 14444 ; 33501 ; 23527 ; 263985 ; 6526]. Repeat parts *(b)-(d)* for this new demand vector.

[1]Wassily W. Leontief, "The Structure of the U.S. Economy," *Scientific American*, April 1965, pp. 30-32.
[2]Use e_8 to extract the last column of the RREF matrix in part (a) and compare this to your answer.

For Week 11: MATLAB #3 - This exploration has **three parts**.

Part 0: NOTE: This part should be handwritten. Create two **easy** or **medium difficulty** exam or quiz style problems using any of the concepts from the previous two weeks (check the schedule to determine the topics). You can pick only one topic, or one problem for each topic; it's totally up to you. You can write any combination of a true/false problem, a possible/impossible, an example construction, a computational problem, or two of the same kind of problem, but your two problems must be using only concepts taught in this course and must not be a problem on one of the practice/sample quizzes/exams. You must also **solve** your problem and state the solution, with a few words of justification, but you do not need to provide an elaborate or detailed solution. Your grade for this portion of the exploration will be based on how well you follow the directions in this paragraph. Don't over think it. Two simple problems will do.

Part 1: Suppose A is a 3×3 matrix with the following eigenvectors and eigenvalues.

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \text{ with eigenvalue } \lambda = 1,$$

$$\vec{v}_2 = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}, \text{ with eigenvalue } \lambda = 0.5,$$

$$\vec{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \text{ with eigenvalue } \lambda = 0.5,$$

(a) Write $\vec{x}$ in the coordinates of the basis $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$.

$$\vec{x} = \begin{bmatrix} 7 \\ 5 \\ 4 \end{bmatrix}$$

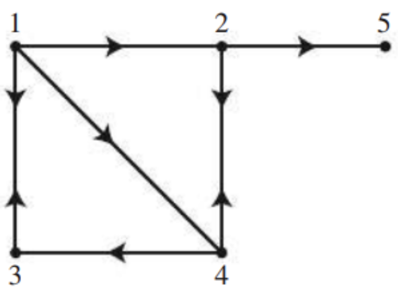
(b) Find $A^k\vec{x}$ (in the standard coordinates) and the coordinates of $A^k\vec{x}$ in the basis $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ for $k = 1, 2, 3, 4, 5$.

(c) Find $\lim_{k\to\infty} A^k\vec{x}$, and compare your answer to the vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$. Comment on why the limit is what it is.

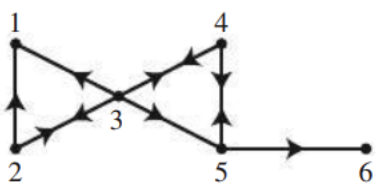
Part 2: Page L10-23 in the online textbook, **first** solve problems 25 and 26. Make a comment on the meaning in the context of google matrices of $G^k(3,2)$ the entry of G^k in row 3 and column 2, and the meaning of $G^k(3,\cdot)$ the 3rd row of G^k.

In Exercises 25 and 26, consider a set of webpages hyperlinked by the given directed graph. Find the Google matrix for each graph and compute the PageRank of each page in the set.

25.



26.



Then, for each google matrix G, find the smallest k such that every entry of G^k is within four significant digits to Π, as in Theorem 1 (Page L10-18). Interpret the meaning in context of the value of $\lim_{k\to\infty} G^k(3,2)$ and $\lim_{k\to\infty} G^k(3,\cdot)$.

THEOREM 1

If P is a regular $m \times m$ transition matrix with $m \geq 2$, then the following statements are all true.

a. There is a stochastic matrix Π such that $\lim_{n\to\infty} P^n = \Pi$.

b. Each column of Π is the same probability vector $\mathbf{q}$.

c. For any initial probability vector $\mathbf{x}_0$, $\lim_{n\to\infty} P^n \mathbf{x}_0 = \mathbf{q}$.

d. The vector $\mathbf{q}$ is the unique probability vector which is an eigenvector of P associated with the eigenvalue 1.

e. All eigenvalues λ of P other than 1 have $|\lambda| < 1$.