MATLAB Exploration #2 for MATH 1554

For each MATLAB assignment, follow the step-by-step formatting guidelines we provided. You will be graded on completeness, following directions, proper usage of comments, and overall readability of your code and published .pdf submission. We recommend `format bank`

For Week 8: MATLAB #2 - This exploration has **two parts**. (See following page for the Markov exploration)

Part 1: *Basis of eigenvectors.* Suppose A is a 3×3 matrix with the following eigenvectors and eigenvalues.

$$\vec{v}_1 = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}, \text{ with eigenvalue } \lambda = 1,$$

$$\vec{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \text{ with eigenvalue } \lambda = \frac{-1}{\sqrt{3}},$$

$$\vec{v}_3 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \text{ with eigenvalue } \lambda = 0.$$

(a) Find $[\vec{x}]_{\mathcal{B}}$ in the coordinates of the basis $\mathcal{B} = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$.

$$\vec{x} = \begin{bmatrix} 4 \\ 11 \\ 2 \end{bmatrix}$$

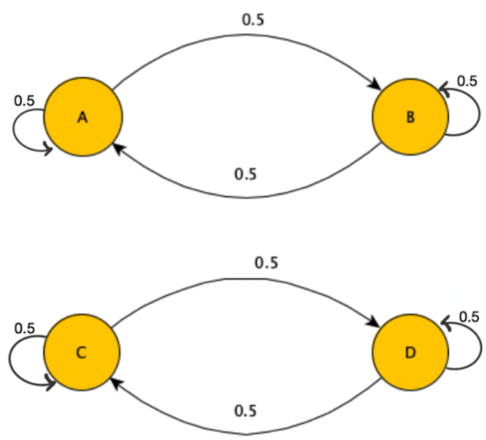
(b) In the comments write $\vec{x}$ as a linear combination of $\vec{v}_1, \vec{v}_2, \vec{v}_3$. Use MATLAB code to check that your linear combination is correct.

(c) For $k = 1, \ldots, 10$, find $[A^k\vec{x}]_{\mathcal{B}}$ the coordinates of $A^k\vec{x}$ in the basis $\mathcal{B} = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$.

(d) In the comments, write $A^k\vec{x}$ as a linear combination of $\vec{v}_1, \vec{v}_2, \vec{v}_3$ for $k = 2, 5, 9, 10$. Check using MATLAB code that your linear combinations are correct.

(e) Find $\lim_{k\to\infty} A^k\vec{x}$ and $\lim_{k\to\infty} [A^k\vec{x}]_{\mathcal{B}}$ in both the standard coordinates and the coordinates in the basis $\mathcal{B} = \{v_1, v_2, v_3\}$. Use comments in your MATLAB code to explain why the limit is what it is. *Hint: if the limit is* DNE *then explain why using the idea of coordinates in the basis* $\mathcal{B}$.

Part 2: *Markov chains.* Consider the transition diagram below.



(a) Find the stochastic matrix P for the transition diagram.

(b) Find the long term trend for the initial distribution $\vec{x}_0 = \begin{bmatrix} 0 \\ .3 \\ 0 \\ .7 \end{bmatrix}$.

(c) Next, find the long term trend for some other initial distribution $\vec{x}_0$ which must satisfy the condition that $x_1 + x_2 \neq 0.3$.

Note: for parts (b) and (c) it is important to use semicolon ; *to suppress output that you don't need to print. We do* NOT *want pages of outputs of* `for` *loops, and any such submission will get a deduction of points based on the 'readability' requirement.*

(d) Find a basis $\mathcal{B} = \{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ of $\mathbb{R}^4$ consisting of eigenvectors for P corresponding to $\lambda_1 = \lambda_2 = 0$ and $\lambda_3 = \lambda_4 = 1$. Find the coordinates of $P^k x_0$ in the basis $\mathcal{B}$ for some value of k between 1 and 5, and some other value of k between 10 and 20. What do you notice? Make a comment about what to expect for the long term trends for any initial $\vec{x}_0$.

(e) At the end, in the comments, answer the following questions:

- **(i)** Does every initial $\vec{x}_0$ have the same long term trend?
- **(ii)** Is P regular? Explain.