Practice Exam 1

1. Find the domain of the function $f(x) = \sqrt{6 - 5x + x^2}$.

2. Solve for x if

$$3x^2 + 15x - 40 = 2.$$

3. Let $f(x) = \frac{4x-2}{\sqrt{x^2+10}}$. Find x such that $f(x) = 2$.

4. Solve the inequality.

$$4x^2 - 40x + 30 \geq -2$$

5. Compute the limit.

$$\lim_{h \to 0} \frac{\sqrt{2(5+h) - 1} - 3}{h}$$

6. Find the derivatives

(a) $(\cos(2x)\sin(4x))'$

(b) $(\tan(\sec x))'$

(c) $(e^{3x-x^2})'$

(d) $(xe^{2x})'$

(e) $\frac{d}{dx}\ln\left(\frac{x}{x^2-2x+1}\right)$

Practice Exam 1

1. Find the domain and range of the functions $f(x) = |x| - 2$, $g(x) = 3^{2-x} + 1$, $h(x) = \frac{3-x}{\sqrt{25-x^2}}$.

2. Find the domain and range of the function

$$f(x) = \begin{cases} \sqrt{-x} & \text{if } -4 \leq x \leq 0 \\ \sqrt{x} & \text{if } 0 < x \leq 4 \end{cases}$$

3. Find $f \circ g$, $g \circ f$, $(f \circ g)(-1)$ and $(g \circ f)(2)$ if $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{\sqrt{x+2}}$.

4. Sketch the graph of

$$f(x) = \begin{cases} -x - 2 & \text{if } -4 \leq x \leq -1 \\ -1 & \text{if } -1 < x \leq 1 \\ x - 2 & \text{if } 1 < x \leq 2 \end{cases}$$

For what values of x is the function $f(x)$ continuous? For what values does the function have a one-sided limit but NOT a two-sided limit?

5. Find the domain of $f(x) = 1 + e^{-\sin(x)}$.

6. Find the largest $\delta > 0$ such that if $|x - 23| < \delta$ then $|f(x) - 4| < \varepsilon$ for $\varepsilon = 1$, where $f(x) = \sqrt{x - 7}$.

7. At what points is the function $f(x) = \frac{x \tan x}{x^2+1}$ continuous? Repeat question for $g(x) = \sqrt{3x - 1}$.

8. Find the limits.

(a) $\displaystyle\lim_{h\to 0^+} \frac{(x+h)^2 - x^2}{h}$

(b) $\displaystyle\lim_{x\to \pi^-} \csc(x)$

(c) $\displaystyle\lim_{x\to 0} \frac{8x}{3\sin x - x}$

(d) $\displaystyle\lim_{x\to \infty} \frac{x^4 + x^3}{12x^3 + 128}$

(e) $\displaystyle\lim_{x\to \infty} \frac{\cos x - 1}{x}$

9. Suppose $\lim_{x\to a} f(x) = -7$ and $\lim_{x\to a} g(x) = 0$. Find the following limits

(a) $\displaystyle\lim_{x\to a} 3f(x) - g(x)$

(b) $\displaystyle\lim_{x\to a} \frac{f(x)}{7 - g(x)}$

(c) $\displaystyle\lim_{x\to a} f(x) \cdot g(x)$

10. Suppose $\lim_{x\to 0} \left(\frac{4 - f(x)}{x}\right) = 1$. Find $\lim_{x\to 0} f(x)$.

1. Find the domain, range, and inverse of the function $f(x) = \dfrac{1-x}{3+x} + 2$. (10 pts.)

2. Let $f(x) = \sqrt{81 - x^2}$ and $g(x) = \frac{1}{x - 4\sqrt{2}}$. Find $g \circ f(x)$ and state the domain of $g \circ f$. (10 pts.)

3. Use the formula $\cos^2(\theta) = \frac{1+\cos(2\theta)}{2}$ to evaluate $\cos^2(\frac{5\pi}{8})$. (10 pts.)

4. Find the value for a that makes the function $f(x)$ continuous on $[-1, 4]$. (10 pts.)

$$f(x) = \begin{cases} -x^2 + 2 & \text{if } -1 \leq x < 2 \\ ax + 3 & \text{if } 2 \leq x < 4 \end{cases}.$$

5. Find the value for x such that $f(x) = e^2$, given that $f(x) = 3 + 6^{-x}$. (10 pts.)

6. In a certain experiment where a particle moves in a straight line, the position of a particle at t seconds is given by the function $f(t) = 1 + t^2$. Find the average speed of the particle in the first three seconds of the experiment. That is, find the average rate of change of $f(t)$ on the interval $[0, 3]$. (10 pts.)

7. Find the limits. (8 pts. each)

(a) $\lim_{x \to 3^-} \dfrac{4x - 12}{9 - x^2}$

(b) $\lim_{x \to \pi^+} \tan(x)$

(c) $\lim_{x \to \infty} \dfrac{3x^3 - x + 3}{4x^2 - \sqrt{x}}$

(d) $\lim_{x \to \infty} \dfrac{3x^3 - 15}{5x^3 + 12x - 4}$

(e) $\lim_{x \to 0} \dfrac{\sin(3x)}{21x}$

1. Solve for x. (10 pts.)

$$3x^2 + 5x + 2 = x^2 - 5$$

2. Find the domain of $f(x) = \sqrt{x^2 + x - 6}$. (10 pts.)

3. Find the equation of the line tangent to the curve $y = f(x)$ at $x = 1$, where (10 pts.)

$$f(x) = \frac{1}{2x + 1}$$

4. Compute the limit. Show all steps for full credit. (10 pts.)

$$\lim_{h \to 0} \frac{\sqrt{4-h}-2}{h}$$

5. Use the formula $(f^{-1})'(b) = \frac{1}{f'(f^{-1}(b))}$ to find $(f^{-1})'(9)$ where $f(x) = x^3 + 1$. Note that $f(2) = 9$. (10 pts.)

6. Suppose $f(2) = 3$, $g(3) = 4$, and that $f'(2) = -1$ and $g'(3) = 2$. Find $(g \circ f)'(2)$. *Hint:* by the chain rule we know that $(g \circ f)'(x) = (g(f(x)))' = g'(f(x)) * f'(x)$. (10 pts.)

7. Find the derivative of the function. (5 pts. each)

(a) $f(x) = \pi x - e$

(b) $g(x) = ex + \ln(x)$

(c) $h(x) = \cos(\sin(x))$

(d) $i(x) = \sec^2(x)$

(e) $j(x) = 4\sqrt{1-x}$

(f) $k(x) = (1-x)(1+x)^2$

(g) $l(x) = xe^{2x}$

(h) $m(x) = \ln\left(\frac{2x}{3-x}\right)$