Calc I and Algebra Review

1. Evaluate $(4^3)^{\frac{1}{2}} \cdot (4^3)^{\frac{1}{2}}$ (5 pts.)

2. Write the standard form of the equation of the circle with radius $r = 3$ and center $(2, -1)$. (5 pts.)

3. Let $f(x) = \sqrt{x-4}$ and $g(x) = 2 - 3x$. Find $(f \circ g)(x)$ and $g \circ g(x)$. Simplify your answer for full credit. (5 pts.)

4. Compute $\sin(\theta)$, $\cos(\theta)$, and $\tan(\theta)$ for $\theta = 135\,^\circ$. (5 pts.)

5. For these problems use $f(x) = x^2 - 4x + 3$.

(a) Find the vertex form of the quadratic function $y = f(x)$. (6 pts.)

(b) Find the factor form of the function $y = f(x)$. (6 pts.)

(c) Graph the quadratic function $y = f(x)$. Be sure to include in your graph the labels for the x-intercepts, y-intercept, and vertex. (3 pts.)

6. Find the inverse of the function $f(x) = \dfrac{2}{3x+1}$, and give the domain and range of f. (7 pts.)

7. Let $f(x) = \sqrt{3x-2}$. Find the average rate of change of $f(x)$ as x changes from 2 to 6. (7 pts.)

8. Simplify the expression $\dfrac{\frac{3}{x+h} - \frac{3}{x}}{h}$. (6 pts.)

9. Evaluate. (5 pts.)

$$\log_2(4^3)$$

10. Write the equation in exponential form, then find the value of y. (5 pts.)

$$\log_5\left(\frac{1}{125}\right) = y$$

11. Find the horizontal and vertical asymptotes of the function $f(x) = \dfrac{x^2 - 2x + 1}{x^2 - 25}$. (5 pts.)

12. Graph the function $y = 3\sec(2x)$, over two periods. (5 pts.)

13. Find all solutions to the equation in the interval $[0, 2\pi)$. (10 pts.)

$$(\csc x - 2)(\cot x + 1) = 0$$

14. Find the equation of the line tangent to the curve $y = \sqrt{3x + 4}$ at $x = 4$. (10 pts.)

15. Use the definition of the derivative to compute $f'(2)$ where $f(x) = \sqrt{2x}$. *You must use the* ***definition*** *of the derivative for full credit.* (10 pts.)

16. Compute limits. Express your answer as a value or one of $+\infty$ DNE, $-\infty$ DNE, or DNE. (5 pts. each)

(a) $\lim_{x \to \frac{\pi}{2}^+} \tan(x)$

(b) $\lim_{x \to \frac{\pi}{3}} \csc(3x)$

(c) $\lim_{x \to 3^-} \dfrac{3x - 9}{x^2 - 9}$

(d) $\lim_{x \to \infty} \dfrac{4x^2 - 3x + 2}{(3x + 1)(2x - 3)}$

(e) $\lim_{x \to 0^+} x \ln(x)$

17. A box is to be constructed using two types of material. The material used to build the top and bottom of the box cost \$10/ft^2 and the material used to build the sides cost \$6/ft^2. If the box must have a volume of 50ft^3 determine the dimensions of the box which will minimize the cost and state the minimum cost of the box. (12 pts.)

18. A right cylindrical tank is filled with water. The tank stands upright and has a radius of 20 cm. How fast does the height of the water in the tank drop when the water is being drained at 25 cm^3/sec? (12 pts.)

19. Compute the derivative. *Continued on next page* (6 pts. each)

(a) $f(x) = (x^2 - 5)(x^3 - 2x + 3)$

(b) $f(x) = \left(\frac{x-1}{x+3}\right)^3$

(c) $f(x) = \left(e^{\sin(2x)} - \pi^2\right)^2$

(d) $f(x) = \ln\left(\frac{\sqrt{3x-1}}{x}\right)$

(e) $f(x) = \sec(\tan(x))$

(f) $f(x) = \frac{\sin^2(x)+\cos^2(x)}{\sin^2(x)}$

20. Integrate. (5 pts. each)

(a) $\int 3x^2 - 4x + \pi^2 - e^{2x} + 4 \, dx$

(b) $\int \left(\frac{1}{\cos 3x}\right)^2 \, dx$