Math 1711 **Exam 3** Spring 2014

Instructor: Sal Barone (A)

Name: KEY

GT username: _____

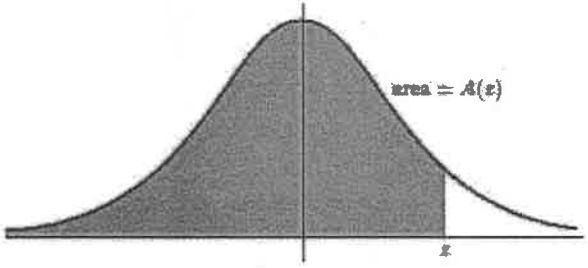
Circle your TA/section: (D1) Ashley (D2) Kayla (D3) Alyssa (D4) Aileen

1. No books or notes are allowed.
2. You may use ONLY NON-GRAPHING and NON-PROGRAMABLE scientific calculators. All other electronic devices are not allowed.
3. Show all work to receive full credit.
4. Write your answers in the box provided.
5. Good luck!

Page	Max. Possible	Points
1	22	
2	26	
3	28	
4	24	
Total	100	

The Normal Distribution

z	$A(z)$	z	$A(z)$	z	$A(z)$
−4.00	.0000	−1.25	.1056	1.50	.9332
−3.75	.0001	−1.00	.1587	1.75	.9599
−3.50	.0002	−.75	.2266	2.00	.9772
−3.25	.0006	−.50	.3085	2.25	.9878
−3.00	.0013	−.25	.4013	2.50	.9938
−2.75	.0030	0	.5000	2.75	.9970
−2.50	.0062	.25	.5987	3.00	.9987
−2.25	.0122	.50	.6915	3.25	.9994
−2.00	.0228	.75	.7734	3.50	.9998
−1.75	.0401	1.00	.8413	3.75	.9999
−1.50	.0668	1.25	.8944	4.00	1.0000



$A(z)$ is the area under the standard normal curve to the left of a normally distributed random variable z.

1. A fair four sided die is a die that has the numbers 1, 2, 3, and 4 and each is equally likely to be cast (it looks like a pyramid). Two fair four sided die are cast and the sum of the numbers shown is recorded. Find the expected value of this experiment. (12 pts.)

k	$Pr(X=k)$
2	1/16
3	2/16
4	3/16
5	4/16
6	3/16
7	2/16
8	1/16

$$E(X) = 2*\tfrac{1}{16} + 3*\tfrac{2}{16} + 4*\tfrac{3}{16} + 5*\tfrac{4}{16} + 6*\tfrac{3}{16} + 7*\tfrac{2}{16} + 8*\tfrac{1}{16}$$

$$= 80/16 = \boxed{5}$$

2. Suppose X is a normally distributed random variable with $\mu = 15$ and $\sigma = 4$. Find $\Pr(7 \leq X \leq 20)$ and $Pr(X \geq 20)$. (10 pts.)

$$Z_1 = \frac{20-15}{4} = 1.25$$

$$Z_2 = \frac{7-15}{4} = -2$$

$$Pr(-2 \leq Z \leq 1.25) = .8944 - .0228 = \boxed{87.16\%}$$

$$Pr(Z \geq 1.25) = 1 - .8944 = \boxed{10.56\%}$$

3. A promoter is considering buying insurance for an outdoor concert. Insurance costs \$5,000 and pays \$25,000 if it rains. If it doesn't rain, the concert will earn the promoter \$30,000. Suppose the promoter calculates his expected earnings in each case of buying or not buying insurance after seeing the weather report. If he decides to buy insurance, what is the lowest chance of rain that he could have seen on the weather report? (16 pts.)

X_1 = profit if buys insurance

X_2 = profit if does NOT buy insurance

p = chance of rain

$$E(X_1) = 20p + 25q$$

$$E(X_2) = 0p + 30q$$

$$E(X_1) \geq E(X_2) \implies 20p + 25q \geq 0p + 30q$$

$$\implies 20p \geq 5q = 5(1-p)$$

$$\implies 25p \geq 5 \implies p \geq \frac{5}{25} = 20\%$$

4. The lengths of six caterpillars in my backyard are recorded below (in millimeters). Calculate the mean, sample variance, and standard deviation for the following data: (10 pts.)

20, 24, 28, 28, 30, 32.

$$\mu = \frac{20 + 24 + 28 + 28 + 30 + 32}{6} = 27$$

$$\sigma^2 = \frac{(20-27)^2 + (24-27)^2 + 2\cdot(28-27)^2 + (30-27)^2 + (32-27)^2}{5}$$

$$\sigma^2 = \frac{49 + 9 + 2 + 9 + 25}{5} = \frac{94}{5} = 18.8$$

$$\sigma = 4.34$$

$\mu = 27$

$\sigma^2 = 18.8$

$\sigma = 4.34$

5. An experiment consists of 30 binomial trials, each with a 20% chance of success.

(a) Find the probability of exactly 5 successes (do not estimate). (8 pts.)

$$Pr(X=5) = \binom{30}{5} \cdot (.20)^5 \cdot (.80)^{25}$$

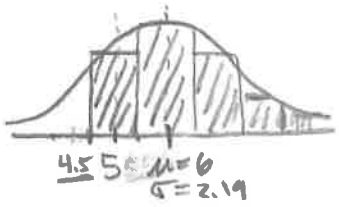
$$= (142506) \cdot (1.20E-6) = .17227$$

$$\approx \boxed{17.2\%}$$

(b) Estimate the probability of at least 5 successes. (12 pts.)

$$\mu = np = 6$$

$$\sigma = \sqrt{npq} = \sqrt{4.8} = 2.19$$



$$Pr(X \geq 4.5) = Pr(Z \geq -.684) \approx 1 - .2266 = .7734$$

$$\frac{4.5-6}{2.19} = -.684$$

$$\boxed{77.34\%}$$

6. An experiment consists of n binomial trials, each with a probability of 15% of success. If the expected value of the experiment is 45, then what is n? (8 pts.)

$$\mu = E(X) = np$$

$$45 = n \cdot 0.15$$

$$n = \frac{45}{0.15} = \boxed{300}$$

7. A committee of three people is to be selected at random from a council consisting of five men and three women. What is the expected number of women on the committee? (12 pts.)

X = number of women.

total # ways to pick 3 from 8

$\binom{8}{3} = 56$

k	$\Pr(X=k)$
0	$\binom{5}{3}/56 = 10/56$
1	$\binom{5}{2}\cdot\binom{3}{1}/56 = 30/56$
2	$\binom{5}{1}\cdot\binom{3}{2}/56 = 15/56$
3	$\binom{5}{0}\cdot\binom{3}{3}/56 = 1/56$

$$E(X) = 0*\frac{10}{56} + 1*\frac{30}{56} + 2*\frac{15}{56} + 3*\frac{1}{56}$$

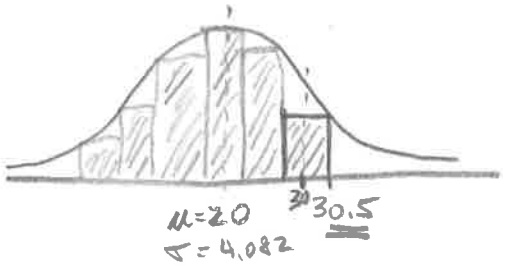
$$E(X) = \frac{63}{56} = \boxed{1.125}$$

8. Estimate the probability of observing at most 30 sixes in 120 rolls of a fair die. (12 pts.)

Chance of a "six"

$$\mu = np = 120 * \frac{1}{6} = 20$$

$$\sigma = \sqrt{npq} = \sqrt{16.6667} = 4.08248$$



$$\Pr(X \le 30.5) = \Pr(Z \le 2.572)$$

$$\approx \boxed{99.38\%}$$