Math 1711 **Exam 4** Spring 2014

Instructor: Sal Barone (A)

Name: KEY

GT username: _____

Circle your TA/section: (D1) Ashley (D2) Kayla (D3) Alyssa (D4) Aileen

1. No books or notes are allowed.
2. You may use ONLY NON-GRAPHING and NON-PROGRAMABLE scientific calculators. All other electronic devices are not allowed.
3. Show all work to receive full credit.
4. Good luck!

Page	Max. Possible	Points
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1. Give an example of two 2×2 matrices A and B which satisfy $AB = BA$. (10 pts.)

$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$AB = BA$ since $AB = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$BA = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

2. Matrix multiply AB or say 'undefined' where (10 pts.)

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 \\ -1 & -1 \\ 2 & 0 \end{bmatrix}.$$

$$AB = \underset{2\times 3}{\begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \end{bmatrix}} \underset{3\times 2}{\begin{bmatrix} 1 & 0 \\ -1 & -1 \\ 2 & 0 \end{bmatrix}} = \underset{2\times 2}{\begin{bmatrix} 0 & -1 \\ 3 & 1 \end{bmatrix}}$$

3. Find the value of a for which the following system has infinitely many solutions: (8 pts.)

$$\begin{cases} 4x - 2y = 1 \\ -2x + y = a \end{cases}$$

$$\left[\begin{array}{cc|c} 4 & -2 & 1 \\ -2 & 1 & a \end{array}\right] \sim \left[\begin{array}{cc|c} 0 & 0 & 2a+1 \\ -2 & 1 & a \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -1/2 & -a/2 \\ 0 & 0 & 2a+1 \end{array}\right]$$

If $a = -1/2$ then the system has infinitely many solutions

If $a \neq -1/2$ the system has no solutions.

4. Given

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

solve the matrix equation $AX = B$ by finding the inverse of A. (12 pts.)

$$A^{-1} = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}^{-1} = \frac{1}{0-(-2)} \begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1/2 & 1/2 \end{bmatrix}$$

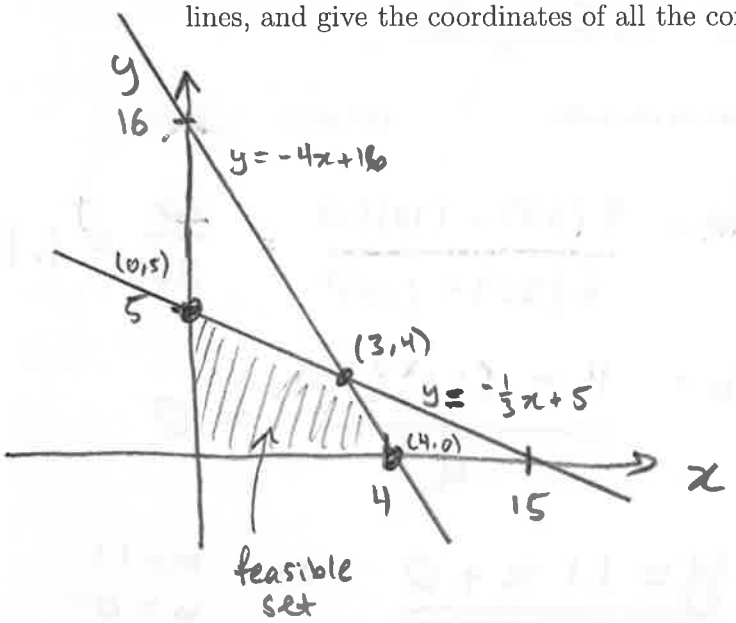
$$X = A^{-1}B = \begin{bmatrix} 0 & -1 \\ 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 3/2 \end{bmatrix}$$

$$X = \begin{bmatrix} -1 \\ 3/2 \end{bmatrix} \quad \text{or} \quad \begin{array}{l} x = -1 \\ y = 3/2 \end{array}$$

5. Find the maximum of the objective function $x + 2y$ over the feasible set defined by the following system of inequalities:

$$\begin{cases} y \geq \frac{-1}{3}x + 5 \\ y \geq -4x + 16 \\ x \geq 0,\ y \geq 0 \end{cases}$$

(a) Draw an accurate picture of the feasible set, label the axes, label the lines, and give the coordinates of all the corners. (15 pts.)



$y = 4$ $\quad -4x + 16 = \frac{-1}{3}x + 5$

$11 = 11/3\, x$

$x = 3$

$y = -4(3) + 16$

$y = -12 + 16$

$y = 4$

(b) What is the maximum of ~~the~~ $x + 2y$ on the feasible set? (5 pts.)

$(0,5)$ $\quad x + 2y \longrightarrow 0 + 2(5) = 10$

$(4,0)$ $\quad x + 2y \longrightarrow 4 + 0 = 4$

$(3,4)$ $\quad x + 2y \longrightarrow 3 + 2(4) = 11$

Maximum of 11 at (3,4)

6. The table below lists widget sales of Company X (in thousands) over the last four years. Use a line of best fit to approximate the amount of widgets that Company X should expect to sell in 2015.

	2011	2012	2013	2014	2015
Profit (mil.)	1	3	2	5	??

(a) Use the formulas

$$m = \frac{N \cdot \sum xy - \sum x \cdot \sum y}{N \cdot \sum x^2 - (\sum x)^2} \qquad b = \frac{\sum y - m \cdot \sum x}{N}$$

to find the line of best fit for the data in the table. (12 pts.)

x	y	xy	x^2
1	1	1	1
2	3	6	4
3	2	6	9
4	5	20	16
10	11	33	30

$$m = \frac{4(33) - (10)(11)}{4(30) - (10)^2} = \frac{22}{20} = 1.1$$

$$b = \frac{11 - (1.1)(10)}{4} = 0$$

$y = 1.1x + 0$ or $m = 1.1$, $b = 0$

(b) Use your answer from above to estimate the number of widgets that Company X should expect to sell in 2015 (and don't forget your units!). (8 pts.)

$y = 1.1x$

$x = 5$ $\quad y = 1.1 * 5 = 5.5$

5,500 widgets

or 5.5 thousand widgets

7. True and False questions (5 pts. each)

(a) If the augmented matrix A represents a system of linear equations and the reduced row echelon form of A is $\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{array}\right]$, then the system that A represents has infinitely many solutions. TRUE **FALSE**

False, unique solution since no free variables

(b) If A and B are square 3×3 matrices, then $AB = BA$. TRUE **FALSE**

AB does not always equal BA

(c) If I is the 3×3 identity matrix and B is a 2×3 matrix, then BI is not defined and $IB = B$. TRUE **FALSE**

BI is defined and equal to B

IB is not defined

(d) The following matrix is in reduced row echelon form. $\left[\begin{array}{cccc} 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$ **TRUE** FALSE

① leading 1's ✓

② column entries above/below leading 1's are zero ✓

③ zero rows come last ✓

④ leading 1's are to the right of leading ones above ✓