

Practice Exam 4

1. Find the value of a for which the corresponding system of linear equalities has no solutions:

$$A = \left[\begin{array}{cc|c} 1 & -1 & 3 \\ 3 & a & 1 \end{array} \right]$$

For which values of a is there a unique solution? Is there any value of a for which there are infinitely many solutions to the system?

2. Given

$$A = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 1 & 2 \\ -1 & -2 \\ 2 & 0 \end{bmatrix}$$

find the rref of each of A , B , C , D and find each of the products AB , BA , AC , CA , AD , DA , BC , CB ,... etc., or say undefined.

3. Let A, B be as in problem #2. Find A^{-1} . Find the solution X to the matrix equation $AX = B$ using the inverse of A .
4. Write down a 3×3 matrix and find its inverse using the Gauss-Jordan Method. It may take a few tries to find one that is 'not too hard'. One that is easy, about the right difficulty for the test (you should be able to do it in < 5 minutes) is

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

5. Use the equation $X = (I - A)^{-1}D$ to find the solution to the input-output analysis problem: A simplified economy consists of the two sectors Manufacturing and Energy. The input-output matrix is

$$\begin{array}{cc} & \begin{matrix} M & E \end{matrix} \\ \begin{matrix} M \\ E \end{matrix} & \begin{bmatrix} .3 & .1 \\ .2 & .25 \end{bmatrix} \end{array}.$$

How many cents of manufacturing are required to produce \$1 of energy? Which sector requires the greatest amount of energy to produce \$1 of output? What should the production rates be in order for the economy to produce \$3 million of manufacturing and \$5 million of energy (in surplus).

6. Use the equations

$$m = \frac{N \cdot \sum xy - \sum x \cdot \sum y}{N \cdot \sum x^2 - (\sum x)^2} \quad b = \frac{\sum y - m \cdot \sum x}{N}$$

to find the line of best fit for the points (1, 2), (2, 4), (3, 3), (4, 4).

7. Use your answer to the previous problem to answer the question: The table below shows the profit of a company over the last four years. What should the company expect in profit in 2015?

	2011	2012	2013	2014	2015
Profit (mil.)	2	4	3	4	??

8. Consider the following linear inequality constraints:

$$\begin{cases} 6x + 3y \leq 96 \\ x + y \leq 18 \\ 2x + 6y \leq 72 \\ x \geq 0, \quad y \geq 0 \end{cases}$$

Graph the feasible set. Find numbers x and y which maximize the objective function $80x + 70y$ and which satisfy all the above constraints simultaneously.