Practice Exam 4

1. Find the value of a for which the corresponding system of linear equalities has no solutions:

$$A = \left[\begin{array}{cc|c} 1 & -1 & 3 \\ 3 & a & 1 \end{array}\right]$$

For which values of a is there a unique solution? Is there any value of a for which there are infinitely many solutions to the system? $a = -3$, $a \neq -3$, No.

2. Given

$$A = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & 2 \\ -1 & -2 \\ 2 & 0 \end{bmatrix}$$

find the rref of each of A, B, C, D and find each of the products AB, BA, AC, CA, AD, DA, BC, CB,... etc., or say undefined. `http://www.bluebit.gr/matrix-calculator/matrix_multiplication.aspx`

3. Let A, B be as in problem #2. Find A^{-1}. Find the solution X to the matrix equation $AX = B$ using the inverse of A. $A^{-1} = \begin{bmatrix} \frac{3}{7} & \frac{-1}{7} \\ \frac{1}{7} & \frac{2}{7} \end{bmatrix}$, $X = \begin{bmatrix} \frac{8}{7} \\ \frac{5}{7} \end{bmatrix}$.

4. Write down a 3×3 matrix and find its inverse using the Gauss-Jordan Method. It may take a few tries to find one that is 'not too hard'. One that is easy, about the right difficulty for the test (you should be able to do it in < 5 minutes) is

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}. \quad A^{-1} = \begin{bmatrix} -1 & 2 & -3 \\ 1 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

5. Use the equation $X = (I - A)^{-1}D$ to find the solution to the input-output analysis problem: A simplified economy consists of the two sectors Manufacturing and Energy. The input-output matrix is

$$\begin{array}{c} \\ M \\ E \end{array} \begin{array}{c} \begin{array}{cc} M & E \end{array} \\ \begin{bmatrix} .3 & .1 \\ .2 & .25 \end{bmatrix} \end{array}.$$

How many cents of manufacturing are required to produce \$1 of energy? Which sector requires the greatest amount of energy to produce \$1 of output? What

should the production rates be in order for the economy to produce \$3 million of manufacturing and \$5 million of energy (in surplus). .1, energy, and they should produce 5.46 m. of manufacturing and 8.12 m. of energy.

6. Use the equations

$$m = \frac{N \cdot \sum xy - \sum x \cdot \sum y}{N \cdot \sum x^2 - (\sum x)^2} \qquad b = \frac{\sum y - m \cdot \sum x}{N}$$

to find the line of best fit for the points $(1, 2), (2, 4), (3, 3), (4, 4)$. $y = .5x + 2$

7. Use your answer to the previous problem to answer the question: The table below shows the profit of a company over the last four years. What should the company expect in profit in 2015?

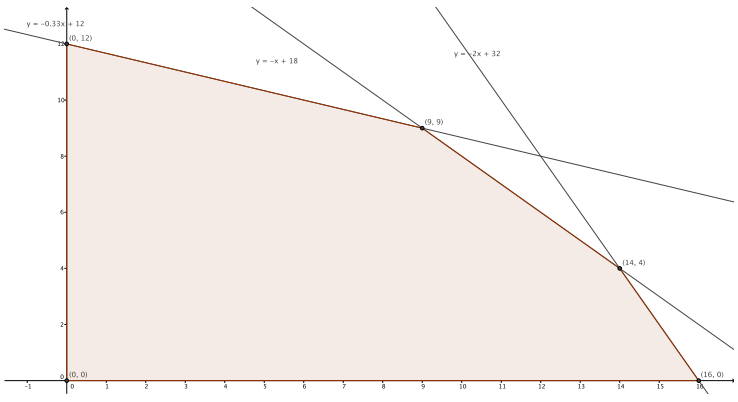
	2011	2012	2013	2014	2015
Profit (mil.)	2	4	3	4	??

4.5 thousand profit expected in 2015

8. Consider the folowing linear inequality constraints:

$$\begin{cases} 6x + 3y \leq 96 \\ x + y \leq 18 \\ 2x + 6y \leq 72 \\ x \geq 0, \quad y \geq 0 \end{cases}$$

Graph the feasible set. Find numbers x and y which maximize the objective function $80x + 70y$ and which satisfy all the above constraints simultaneously.



$x = 14,\ y = 4$