Math 1711 **Exam 4** Spring 2014

Instructor: Sal Barone (B)

Name: KEY

GT username: ______________________

Circle your TA/section: (D1) Ashley (D2) Kayla (D3) Alyssa (D4) Aileen

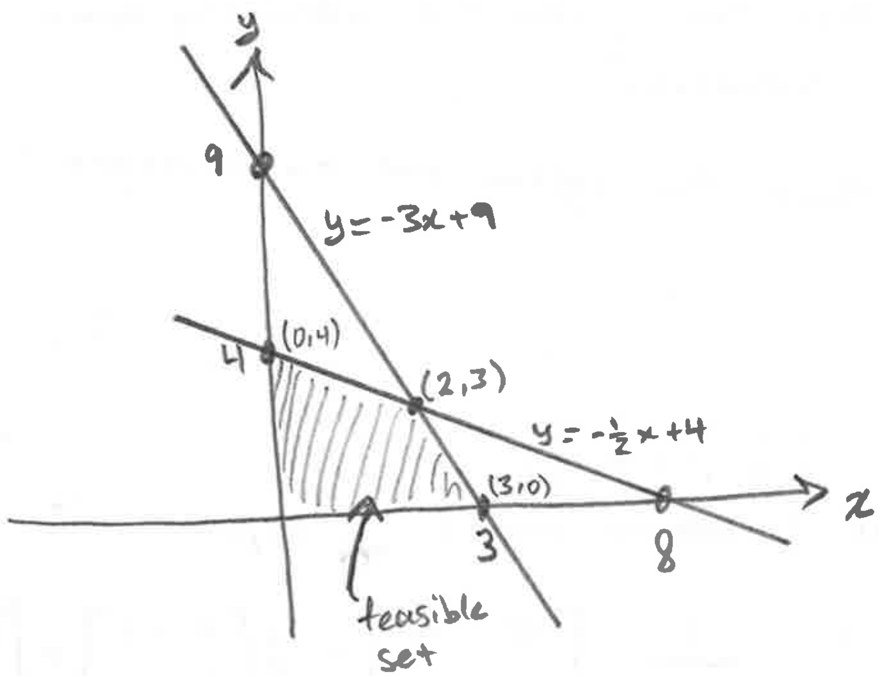
1. No books or notes are allowed.
2. You may use ONLY NON-GRAPHING and NON-PROGRAMABLE scientific calculators. All other electronic devices are not allowed.
3. Show all work to receive full credit.
4. Good luck!

Page	Max. Possible	Points
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1. Find the maximum of the objective function $x + 3y$ over the feasible set defined by the following system of inequalities:

$$\begin{cases} y \geq \frac{-1}{2}x + 4 \\ y \geq -3x + 9 \\ x \geq 0, \; y \geq 0 \end{cases}$$

(a) Draw an accurate picture of the feasible set, label the axes, label the lines, and give the coordinates of all the corners. (15 pts.)



$y = y$

$-\frac{1}{2}x + 4 = -3x + 9$

$\frac{5}{2}x = 5$

$x = 2$

$y = -3(2) + 9$

$y = -6 + 9$

$y = 3$

(b) What is the maximum of the $x + 3y$ on the feasible set? (5 pts.)

$(0,4) \quad x + 3y \longrightarrow 0 + 3(4) = 12$

$(3,0) \quad x + 3y \longrightarrow 3 + 0 = 3$

$(2,3) \quad x + 3y \longrightarrow 2 + 3(3) = 11$

maximum of 12 occurs at (0,4)

2. Find the value of a for which the following system has infinitely many solutions: (8 pts.)

$$\begin{cases} 6x - 2y = -1 \\ -3x + y = a \end{cases}$$

$$\left[\begin{array}{cc|c} 6 & -2 & -1 \\ -3 & 1 & a \end{array}\right] \sim \left[\begin{array}{cc|c} 0 & 0 & 2a-1 \\ -3 & 1 & a \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -1/3 & -a/3 \\ 0 & 0 & 2a-1 \end{array}\right]$$

if $\underline{a = 1/2}$ then the system has infinitely many solutions

if $a \neq 1/2$ then the system has no solutions.

3. Given

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

solve the matrix equation $AX = B$ by finding the inverse of A. (12 pts.)

$$A^{-1} = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}^{-1} = \frac{1}{0-(-2)} \begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1/2 & 1/2 \end{bmatrix}$$

$$X = A^{-1}B = \begin{bmatrix} 0 & -1 \\ 1/2 & 1/2 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 3/2 \end{bmatrix}$$

$$\underline{X = \begin{bmatrix} -1 \\ 3/2 \end{bmatrix}} \quad \text{or} \quad \begin{array}{l} x = -1 \\ y = 3/2 \end{array}$$

4. Give an example of two 2×2 matrices A and B which satisfy $AB = BA$. (10 pts.)

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Then $AB = BA$ since

$$AB = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \text{ and } BA = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}.$$

5. Matrix multiply AB or say 'undefined' where (10 pts.)

$$A = \begin{bmatrix} 2 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 2 & -1 \end{bmatrix}.$$

$$\underset{2\times 3}{\begin{bmatrix} 2 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix}} \underset{3\times 2}{\begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 2 & -1 \end{bmatrix}} = \underset{2\times 2}{\begin{bmatrix} 1 & 0 \\ 3 & -1 \end{bmatrix}}$$

6. The table below lists widget sales of Company X (in thousands) over the last four years. Use a line of best fit to approximate the amount of widgets that Company X should expect to sell in 2015.

	2011	2012	2013	2014	2015
Profit (mil.)	2	3	3	5	??

(a) Use the formulas

$$m = \frac{N \cdot \sum xy - \sum x \cdot \sum y}{N \cdot \sum x^2 - (\sum x)^2} \qquad b = \frac{\sum y - m \cdot \sum x}{N}$$

to find the line of best fit for the data in the table. (12 pts.)

x	y	xy	x^2
1	2	2	1
2	3	6	4
3	3	9	9
4	5	20	16
10	13	37	30

$$m = \frac{4(37) - (10)(13)}{4(30) - (10)^2} = \frac{18}{20} = .9$$

$$b = \frac{13 - .9(10)}{4} = 1$$

$y = .9x + 1$ or $m = .9$, $b = 1$

(b) Use your answer from above to estimate the number of widgets that Company X should expect to sell in 2015 (and don't forget your units!). (8 pts.)

$x = 5$ $\quad y = .9(5) + 1 = 5.5$

5,500 widgets

or 5.5 Thousand widgets

7. True and False questions (5 pts. each)

(a) The following matrix is in reduced row echelon form. $\begin{bmatrix} 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ TRUE FALSE

(b) If I is the 3×3 identity matrix and B is a 2×3 matrix, then IB is not defined and $BI = B$. TRUE FALSE

(c) If A and B are square 3×3 matrices, then $AB = BA$. TRUE FALSE

(d) If the augmented matrix A represents a system of linear equations and the reduced row echelon form of A is $\left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$, then the system that A represents has a unique solution. TRUE FALSE