Practice Exam 2

1. True or False questions.

F (a) The matrix $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ corresponds to a system of linear equations with ~~infinitely many solutions~~. Unique soln $x=y=z=0$

T (b) Given two mutually exclusive events E and F, we have $P(E \text{ or } F) = P(E) + P(F)$. Since $P(E \cap F) = 0$

T (c) If E and F are independent events then $P(E \text{ and } F) = P(E) \cdot P(F|E)$. always true, in particular when E & F are ind. events.

T (d) If I is the 3×3 identity matrix and A is any 3×3 matrix, then $AI = IA$.

F (e) Roll a die and record the number and let E and F be the following events $E = \{2, 4, 6\}$ and $F = \{1, 3, 5\}$. Then the events E and F are independent. $P(E|F) = 0$ but $P(E) = 1/2$, so not independent

2. Find the matrix product of AB and BA if

$$A = \begin{bmatrix} 2 & 3 \\ -1 & 0 \\ 4 & 1 \end{bmatrix} \quad \text{and } B = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \end{bmatrix}.$$

$AB = \begin{bmatrix} -1 & 4 & 9 \\ -1 & -2 & -3 \\ 3 & 8 & 13 \end{bmatrix}$ &

$BA = \begin{bmatrix} 12 & 6 \\ 2 & -2 \end{bmatrix}$

3. Solve the system of linear equations with augmented matrix A given below. Use elementary row operations to obtain the rref (reduced row echelon form) of A and be precise in your answer. You should assume that the column variables are x, y, z in the usual order.

$$A = \left[\begin{array}{ccc|c} 2 & 0 & -2 & 6 \\ -1 & 2 & 3 & 7 \\ 1 & 2 & 1 & 13 \end{array}\right]$$

$A \sim \left[\begin{array}{ccc|c} 2 & 0 & -2 & 6 \\ -1 & 2 & 3 & 7 \\ -1 & 2 & 3 & 7 \end{array}\right]$ ($-R_1+R_3 \to$) $\sim$ ($-R_2+R_3 \to$) $\left[\begin{array}{ccc|c} 2 & 0 & -2 & 6 \\ -1 & 2 & 3 & 7 \\ 0 & 0 & 0 & 0 \end{array}\right] \sim$ ($\frac{1}{2}R_1$) $\left[\begin{array}{ccc|c} 1 & 0 & -1 & 3 \\ -1 & 2 & 3 & 7 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\sim$ ($R_1+R_2 \to$) $\left(\begin{array}{ccc|c} 1 & 0 & -1 & 3 \\ 0 & 2 & 2 & 10 \\ 0 & 0 & 0 & 0 \end{array}\right) \sim$ ($\frac{1}{2}R_2 \to$) $\left[\begin{array}{ccc|c} 1 & 0 & -1 & 3 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{array}\right]$ z free

$$\boxed{\begin{aligned} x &= 3+z \\ y &= 5-z \\ z &= \text{free} \end{aligned}}$$

4. Consider an experiment where two fair dice are rolled and the sum of the two numbers are recorded. Let X be the sum of the two numbers which appear face up on the dice. Find the expected value and variance of X.

x	$P(X=x)$
2	1/36
3	2/36
4	3/36
5	4/36
6	5/36
7	6/36
8	5/36
9	4/36
10	3/36
11	2/36
12	1/36

$$\mu = E(X) = 2\cdot\tfrac{1}{36} + 3\cdot\tfrac{2}{36} + 4\cdot\tfrac{3}{36} + 5\cdot\tfrac{4}{36} + 6\cdot\tfrac{5}{36} + 7\cdot\tfrac{6}{36} + 8\cdot\tfrac{5}{36} + 9\cdot\tfrac{4}{36} + 10\cdot\tfrac{3}{36} + 11\cdot\tfrac{2}{36} + 12\cdot\tfrac{1}{36} = \frac{252}{36}$$

$$E(X) = \underline{\underline{7}}.$$

$$\sigma^2 = E\left((X-\mu)^2\right) = \frac{(-5)^2(1)+(-4)^2(2)+(-3)^2(3)+(-2)^2(4)+(-1)^2(5)+0\cdot 6 + (1)^2(5)+(2)^2(4)+(3)^2(3)+(4)^2(2)+(5)^2(1)}{36}$$

$$\sigma^2 = 185/36 = \underline{\underline{5.13\overline{8}}}$$

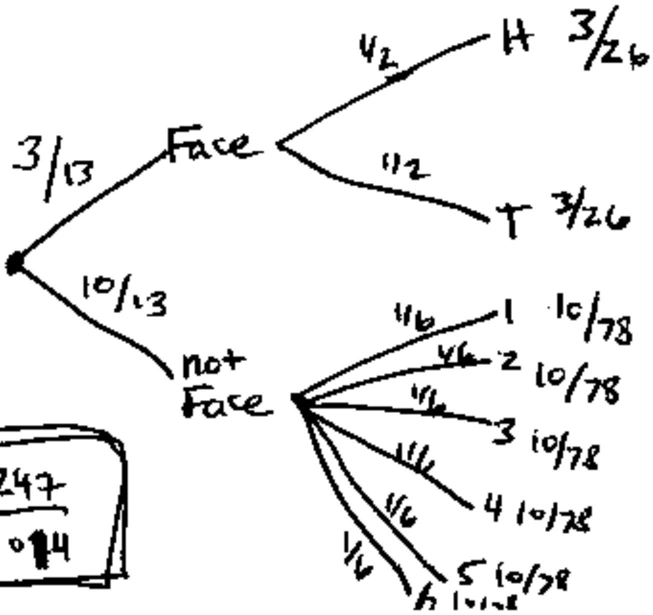
5. Suppose four fair die are rolled. What is the probability that at least one of the die shows either a 1 or a 2?

binomial trial with $n=4$ and $p=1/3$.

$X = \#$ of success so

$$P(X\geq 1) = 1 - P(X=0) = 1 - \left(\tfrac{2}{3}\right)^4 = \underline{\underline{80.247\%}}.$$

6. Consider the following two-stage experiment. First, we draw a card from a 52-card deck. If the card is a face-card then we flip a coin, and if it is not a face card then we roll a die. Find the probability that we end the sequence with a "6" on the die or with a "heads" on the coin.



$P(\text{"6" or "heads" in second stage})$

$= P(\text{"6"} \mid \text{not face}) + P(\text{"heads"} \mid \text{face})$

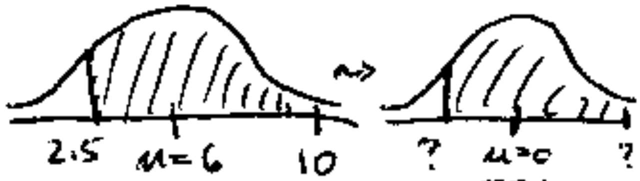
$= 3/26 + \frac{10}{78} = \frac{494}{2028}$

$$\frac{494}{2028} = \boxed{\frac{247}{1014}}$$

7. Let X be a normally distributed continuous random variable with $\mu = 6$ and $\sigma = 2$. Find $P(X \le 5)$ and $P(2.5 \le X \le 10)$.

$Z = \frac{5-6}{2} = -\frac{1}{2}$

table $\rightsquigarrow P(Z \le -\frac{1}{2}) = \boxed{.30854}$



$Z = \frac{2.5-6}{2} = \frac{-3.5}{2} = -1.75$

$Z = \frac{10-6}{2} = 2$

$P(-1.75 \le Z \le 2) = .97725 - .0400$

$= \boxed{.93719}$

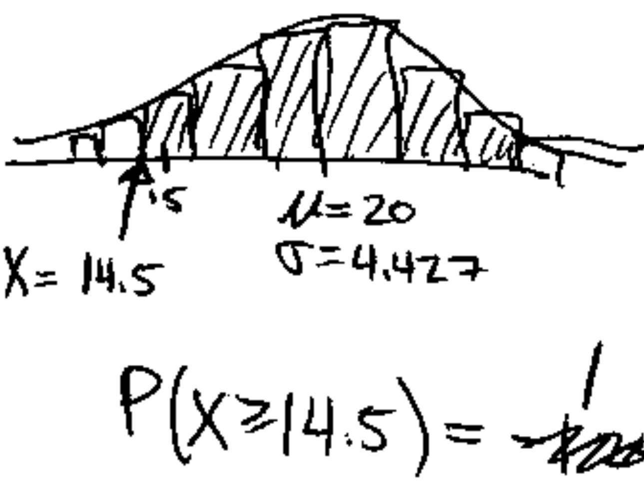
8. A washing machine manufacturer knows that 2% of its machines break down in the first year. Estimate the probability of at least 15 out of 1000 washers breaking down in the first year.

Estimate binomial with $n = 1000$ and $p = 0.02$

by Normal dist with $\mu = n \cdot p = 20$

and $\sigma = \sqrt{npq} = \sqrt{1000(0.02)(0.98)}$

$\sigma = 4.427$



$Z = \frac{14.5-20}{4.427} = -1.242$

$P(X \ge 14.5) = 1 - P(Z \le -1.24)$

$= 1 - .10749$

$= \boxed{.89251}$