

FORMULA SHEET Exam 1

- $\langle u_1, u_2, u_3 \rangle \cdot \langle v_1, v_2, v_3 \rangle = u_1 v_1 + u_2 v_2 + u_3 v_3$

- $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}| |\mathbf{v}| \cos(\theta)$

- $\langle u_1, u_2, u_3 \rangle \times \langle v_1, v_2, v_3 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$

- $|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}| |\mathbf{v}| |\sin(\theta)|$

- $L = \int_a^b |\mathbf{r}'(t)| \, dt$

- $s(t) = \int_{t_0}^t |\mathbf{r}'(\tau)| \, d\tau$

- $\mathbf{T} = \frac{\mathbf{v}}{|\mathbf{v}|} = \frac{d\mathbf{r}}{ds}$

- $\kappa = \left| \frac{d\mathbf{T}}{ds} \right| = \frac{1}{|\mathbf{v}|} \left| \frac{d\mathbf{T}}{dt} \right| = \frac{|\mathbf{v} \times \mathbf{a}|}{|\mathbf{v}|^3}$

- $\mathbf{N} = \frac{1}{\kappa} \frac{d\mathbf{T}}{ds} = \frac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|}$

FORMULA SHEET Exam 2

- Total Derivative: For $\mathbf{f}(x_1, \dots, x_n) = \langle f_1(x_1, \dots, x_n), \dots, f_m(x_1, \dots, x_n) \rangle$

$$D\mathbf{f} = \begin{bmatrix} (f_1)_{x_1} & (f_1)_{x_2} & \cdots & (f_1)_{x_n} \\ (f_2)_{x_1} & (f_2)_{x_2} & \cdots & (f_2)_{x_n} \\ \vdots & \ddots & \cdots & \vdots \\ (f_m)_{x_1} & (f_m)_{x_2} & \cdots & (f_m)_{x_n} \end{bmatrix}$$

- Linearization: Near $\mathbf{a}$, $f(\mathbf{x}) \approx L(\mathbf{x}) = f(\mathbf{a}) + Df(\mathbf{a})(\mathbf{x} - \mathbf{a})$
- Chain Rule: If $h = g(f(\mathbf{x}))$ then $Dh(\mathbf{x}) = Dg(f(\mathbf{x}))Df(\mathbf{x})$
- Implicit Differentiation: If z is implicitly given in terms of x and y by $F(x, y, z) = c$, then $\frac{\partial z}{\partial x} = \frac{-F_x}{F_z}$ and $\frac{\partial z}{\partial y} = \frac{-F_y}{F_z}$.
- Directional Derivative: If $\mathbf{u}$ is a unit vector, $D_{\mathbf{u}}f(P) = Df(P)\mathbf{u} = \nabla f(P) \cdot \mathbf{u}$
- The tangent line to a level curve of $f(x, y)$ at (a, b) is $0 = \nabla f(a, b) \cdot \langle x - a, y - b \rangle$
- The tangent plane to a level surface of $f(x, y, z)$ at (a, b, c) is $0 = \nabla f(a, b, c) \cdot \langle x - a, y - b, z - c \rangle$.
- Hessian Matrix: For $f(x, y)$, $Hf(x, y) = \begin{bmatrix} f_{xx} & f_{yx} \\ f_{xy} & f_{yy} \end{bmatrix}$
- Second Derivative Test: If (a, b) is a critical point of $f(x, y)$ then
 1. If $\det(Hf(a, b)) > 0$ and $f_{xx}(a, b) < 0$ then f has a local maximum at (a, b)
 2. If $\det(Hf(a, b)) > 0$ and $f_{xx}(a, b) > 0$ then f has a local minimum at (a, b)
 3. If $\det(Hf(a, b)) < 0$ then f has a saddle point at (a, b)
 4. If $\det(Hf(a, b)) = 0$ the test is inconclusive

- Area/volume: $\text{area}(R) = \iint_R dA$

- Trig identities: $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$, $\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$

- Average value: $f_{\text{avg}} = \frac{\iint_R f(x, y) dA}{\text{area of } R}$

- Polar coordinates: $x = r \cos(\theta)$, $y = r \sin(\theta)$, $dA = r \, dr \, d\theta$

FORMULA SHEET After Exam 2

- Trig identities: $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$, $\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$
- Volume(D) = $\iiint_D dV$, $f_{avg} = \frac{\iiint_D f(x, y, z) dV}{\text{volume of } D}$ or $\frac{\int_C f(x, y, z) ds}{\text{length of } C}$,
Mass: $M = \iiint_D \delta dV$
- Cylindrical coordinates: $x = r \cos(\theta)$, $y = r \sin(\theta)$, $z = z$, $dV = r dz dr d\theta$
- Spherical coordinates: $x = \rho \sin(\phi) \cos(\theta)$, $y = \rho \sin(\phi) \sin(\theta)$, $z = \rho \cos(\phi)$,
 $dV = \rho^2 \sin(\phi) d\rho d\phi d\theta$
- Substitution for double integrals: If R is the image of G under a coordinate transformation $\mathbf{T}(u, v) = \langle x(u, v), y(u, v) \rangle$ then

$$\iint_R f(x, y) dx dy = \iint_G f(\mathbf{T}(u, v)) |\det D\mathbf{T}(u, v)| du dv.$$