Math 2550 Worksheet Section 12.2

1. Let $A = (1, 1)$, $B = (1, 0)$, $C = (-1, 3)$, and $D = (-2, 2)$. Let $\vec{v} = \overrightarrow{AB} + \overrightarrow{CD}$.
   (a) Find the component form of $\vec{v}$.
   (b) Express $\vec{v}$ in the form of $v_1\hat{\imath} + v_2\hat{\jmath}$.
   (c) Find the magnitude (length) of the $\vec{v}$.
   (d) Find the unit vector in the direction of $\vec{v}$.
2. Let $\vec{u} = \langle 1, 1, -1\rangle$ and $\vec{v} = \langle 2, 0, 3\rangle$.
   (a) Find the component form of $2\vec{u} - \vec{v}$.
   (b) Express $\vec{u}$ as a product of its length and direction.
   (c) Find a vector of magnitude 2 in the direction of $\vec{v}$.
3. Let $A = (-1, 1, 5)$ and $B = (2, 5, 0)$.
   (a) What is the midpoint of line segment AB?
   (b) If $\overrightarrow{AC} = \hat{\imath} + 4\hat{\jmath} - 2\hat{k}$, what is C?

Answer Key

1. (a) $\vec{v} = \langle -1, -2 \rangle$.
   (b) $\vec{v} = -\hat{i} - 2\hat{j}$.
   (c) $|\vec{v}| = \sqrt{5}$.
   (d) $\hat{v} = \frac{1}{\sqrt{5}} \langle -1, -2 \rangle$.
2. (a) $\langle 0, 2, -5 \rangle$.
   (b) $\vec{u} = \sqrt{3} \cdot \langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} \rangle$.
   (c) $\vec{v} = \frac{2}{\sqrt{13}} \cdot \langle 2, 0, 3 \rangle$.
3. (a) $\left(\frac{1}{2}, 3, \frac{5}{2} \right)$.
   (b) $C = (0, 5, 3)$.

Math 2550 Worksheet Section 12.3

1. Let $\vec{v} = \langle 2, -4, \sqrt{5} \rangle$ and $\vec{u} = \langle -2, 4, -\sqrt{5} \rangle$. Compute the following:
   (a) $\vec{v} \cdot \vec{u}$
   (b) the cosine of the angle between $\vec{v}$ and $\vec{u}$.
   (c) $\text{proj}_{\mathbf{v}}\, \mathbf{u}$.
   (d) $(3\vec{v}) \cdot (2\vec{u})$.
2. Are $\vec{u} = 3\hat{\imath} - 2\hat{\jmath}$ and $\vec{v} = 4\hat{\imath} + 6\hat{\jmath}$ orthogonal? Why or why not? Also, sketch these vectors.
3. Suppose that a box on a horizontal floor is being towed at an angle of $30°$ to the right with a force $\vec{F}$ of magnitude 22 newtons.
   (a) Draw a diagram.
   (b) What is the horizontal and vertical components of the force?
   (c) How much work is done by the force $\vec{F}$ if the box is pulled 7 meters?

Answer Key

1. (a) -25.
   (b) $\cos\theta = -1$.
   (c) $\langle -2, 4, -\sqrt{5} \rangle$.
   (d) -150.
2. Yes.
3. (a) N/A
   (b) Horizontal component $= 11\sqrt{3}$ and vertical component $=11$.
   (c) $77\sqrt{3}$.

Math 2550 Worksheet Section 12.4

1. Let $\vec{u} = 2\hat{i} - 2\hat{j} - \hat{k}$ and $\vec{v} = \hat{i} - \hat{k}$. Compute the following:
   (a) $\vec{u} \times \vec{v}$.
   (b) $3\vec{u} \times 2\vec{v}$.
   (c) $\vec{v} \times \vec{u}$.
2. Let $P = (1, -1, 2)$, $Q = (2, 0, -1)$, and $R = (0, 2, 1)$.
   (a) Find the area of the triangle determined by the points P, Q, and R.
   (b) Find a unit vector normal to the plane containing P, Q, and R.
3. Find the volume of the parallelepiped, where four of whose vertices are $A(0, 0, 0)$, $B(1, 2, 0)$, $C(0, -3, 2)$, $D(3, -4, 5)$ such that vertex D does not lie in the same plane as A, B, and C.

Answer Key

1. (a) $2\hat{i} + 1\hat{j} + 2\hat{k}$.

   (b) $12\hat{i} + 6\hat{j} + 12\hat{k}$.

   (c) $-2\hat{i} - \hat{j} - 2\hat{k}$.

2. (a) $2\sqrt{6}$.

   (b) $\frac{1}{\sqrt{6}}\langle 2, 1, 1\rangle$.

3. 5.

Math 2550 Worksheet Section 12.5

1. Find parametric equation for
   (a) the line through point $P = (1, 2, -1)$ and point $Q(-1, 0, 1)$.
   (b) the line through $(0, -7, 0)$ perpendicular to the plane $x + 2y + 2z = 13$.
   (c) the line in which the planes $3x - 6y - 2z = 3$ and $2x + y - 2z = 2$ intersect.
2. How do we know that the points $(1, 1, -1)$, $(2, 0, 2)$, and $(0, -2, 1)$ determine a unique plane? Find the equation of the plane through $(1, 1, -1)$, $(2, 0, 2)$, and $(0, -2, 1)$.
3. Find the distance from the point $(2, 1, 3)$ to the line $x = 2 + 2t$, $y = 1 + 6t$, $z = -3 - 5t$.
4. Find the distance from the point $(2, -3, 4)$ to the plane $x + 2y + 2z = 13$.
5. When will 3 distinct points NOT determine a unique plane? Find 2 planes that are not parallel that both contain the points $P(1, -1, 1)$, $Q(3, 2, 0)$, and $R(5, 5, -1)$.

Answer Key

1. (a) $x = 1 - 2t$, $y = 2 - 2t$, $z = -1 + 2t$.

   (b) $x = t$, $y = -7 + 2t$, $z = 2t$.

   (c) $x = 1 + 14t$, $y = 2t$, $z = 15t$.

2. $7x - 5y - 4z = 6$.

3. $\dfrac{12\sqrt{2}}{\sqrt{13}}$.

4. 3.

5. Think about it.

Math 2550 Worksheet Section 13.1

1. Given the position of a particle in the xy-plane at time t: $\vec{r}(t) = e^t\hat{\imath} + \frac{2}{9}e^{2t}\hat{\jmath}$, $t = \ln 3$,
   (a) find an equation in x and y whose graph is the path of the particle.
   (b) find the particle's velocity and acceleration vectors at the given value of t.
   (c) Sketch the path of the particle and include the particle's velocity and acceleration vectors at the given value of t.
2. Given the position of a particle in the xy-plane at time t: $\vec{r}(t) = (2\cos t)\hat{\imath} + (3\sin t)\hat{\jmath} + 4t\hat{k}$, $t = \pi/2$,
   (a) find the particle's velocity and acceleration vectors.
   (b) write the particle's velocity at the given value of t as the product of its speed and direction.
3. Find the parametric equations for the line that is tangent to the curve

$$\vec{r}(t) = \left\langle \ln t, \frac{t-1}{t+2}, t\ln t \right\rangle, \text{ at } t = 1.$$

Answer Key

1. (a) $y = \frac{2}{9}x^2$, $x > 0$.

   (b) $\vec{v}(t) = \vec{r}'(t) = e^t\hat{\imath} + \frac{4}{9}e^{2t}\hat{\jmath}$ and

   $\vec{a}(t) = \vec{v}'(t) = e^t\hat{\imath} + \frac{8}{9}e^{2t}\hat{\jmath}$

   (c) $\vec{a}(\ln 3) = 3\hat{\imath} + 8\hat{\jmath}$.

   $\vec{v}(\ln 3) = 3\hat{\imath} + 4\hat{\jmath}$.

2. (a) $\vec{v}(t) = (-2\sin t)\hat{\imath} + (3\cos t)\hat{\jmath} + 4\hat{k}$.

   $\vec{a}(t) = (-2\cos t)\hat{\imath} - (3\sin t)\hat{\jmath}$.

   (b) $\vec{v}(\pi/2) = 2\sqrt{5}\left(-\frac{1}{\sqrt{5}}\hat{\imath} + \frac{2}{\sqrt{5}}\hat{k}\right)$.

3. $x = t$, $y = \frac{1}{3}t$, $z = t$.

Math 2550 Worksheet Section 13.2

1. Suppose that $\vec{r}(t)$ satisfies
$$\vec{r}''(t) = -\hat{i} - \hat{j} - \hat{k}, \quad t \geq 0, \qquad \vec{r}'(0) = 5\hat{i}, \qquad \vec{r}(0) = 10\hat{i} + 10\hat{j} + 10\hat{k}.$$
Find $\vec{r}(t)$.

2. A baseball is hit when it is 2.5 ft above the ground. It leaves the bat with an initial velocity of 140 ft/sec at a launch angle of 30°. At the instant the ball is hit, an instantaneous gust of wind blows against the ball, adding a component of $-14\hat{i}$ (ft/sec) to the ball's initial velocity. A 15 ft high fence lies 400 ft from the home plate in the direction of the flight. (Note that gravity, g = 32 ft/sec^2)
   (a) Include an appropriate sketch.
   (b) Find a vector equation for the path of the baseball.
   (c) How high does the baseball go, and when does it reach maximum height?
   (d) Find the range and flight time of the baseball, assuming that the ball is not caught.
   (e) When is the baseball 20 ft high? How far (ground distance) is the baseball from home plate at that height?
   (f) Has the batter hit a home run? Explain.

Answer Key

1. $\vec{r}(t) = (10 + 5t - \frac{1}{2}t^2)\hat{i} + (10 - \frac{1}{2}t^2)\hat{j} + (10 - \frac{1}{2}t^2)\hat{k}$
2. (a) You can do this!

   (b) $\vec{r}(t) = (70\sqrt{3} - 14)t\hat{i} + (2.5 + 70t - 16t^2)\hat{j}$.

   (c) $y_{\text{max}} = 79.0625$ ft., which is reached at $t = 2.1875$ s.

   (d) $t = 4.41$s. 472.94 ft.

   (e) 29 ft and 441 ft.

   (f) Yes.

Math 2550 Worksheet Section 13.3

1. Given $\vec{r}(t) = (6\sin 2t)\hat{i} + (6\cos 2t)\hat{j} + 5t\hat{k}$, $0 \leq t \leq \pi$,
   (a) find the unit tangent vector of $\vec{r}(t)$.
   (b) find the length of the indicated portion of $\vec{r}(t)$.
2. Find the point on the curve
   $$\vec{r}(t) = (5\sin t)\hat{i} + (5\cos t)\hat{j} + 12t\hat{k}$$
   at a distance 26π units along the curve from the point $(0, 5, 0)$ in the direction of increasing arc length.
3. Given $\vec{r}(t) = (2\ln(t+1))\hat{i} + (e^{2t} + t)\hat{j} + (\sin^2(t))\hat{k}$, set up the appropriate integral with limits to find the length of the course from point $A(0, 1, 0)$ to $B(\ln 4, e^2 + 1, \sin^2(1))$.
4. Find the length of the curve
   $$\vec{r}(t) = (\sqrt{2}t)\hat{i} + (\sqrt{3}t)\hat{j} + (1-t)\hat{k}$$
   from $(0, 0, 1)$ to $(\sqrt{2}, \sqrt{3}, 0)$.

Answer Key

1. (a) $\vec{T} = (\frac{12}{13}\cos 2t)\hat{i} - (\frac{12}{13}\sin 2t)\hat{j} + \frac{5}{13}\hat{k}$.

   (b) 13π.

2. $(0, 5, 24\pi)$.

3. $s = \int_0^1 \sqrt{\frac{4}{(t+1)^2} + (2e^{2t}+1)^2 + 4\sin^2 t\cos^2 t}\ dt$.

4. $\sqrt{6}$.

Math 2550 Worksheet Section 13.4

1. Find $\vec{T}$, $\vec{N}$, and κ for
   (a) $\vec{r}(t) = (3\sin t)\hat{\imath} + (3\cos t)\hat{\jmath} + 4t\hat{k}$.
   (b) $\vec{r}(t) = \langle t, \ln\cos t\rangle$, $-\pi/2 < t < \pi/2$.
2. The graph $y = f(x)$ in the xy-plane automatically has parametrization $x = x$ and $y = f(x)$, and the vector formula $\vec{r}(x) = x\hat{\imath} + f(x)\hat{\jmath}$. Use this formula to show that if f is a twice-differentiable function of x, then
$$\kappa(x) = \frac{|f''(x)|}{[1 + (f'(x))^2]^{3/2}}.$$
3. Find $\kappa(x)$ for
   (a) $f(x) = e^x$.
   (b) $f(x) = \sin x$.
4. Determine the maximum curvature for $f(x) = \ln x$.
5. Let $\vec{r}(t) = -(t + (1/t))\hat{\imath} + (2\ln t)\hat{\jmath}$, $e^{-5} \le t \le e^5$.
   (a) Find the radius curvature at $t = 1$.
   (b) Find $\vec{N}$ at $t = 1$,
   (c) Find the center of the circle of curvature at $t = 1$.
   (d) Find the equation for the circle of curvature at $t = 1$.

Answer Key

1. (a) $\overrightarrow{T} = \frac{3\cos(t)}{5}\hat{i} - \frac{3\sin(t)}{5}\hat{j} + \frac{4}{5}\hat{k}$, $\overrightarrow{N} = -\sin(t)\hat{i} - \cos(t)\hat{j}$, and $\kappa = \frac{3}{25}$

   (b) $\overrightarrow{T} = \cos(t)\hat{i} - \sin(t)\hat{j}$, $\overrightarrow{N} = -\sin(t)\hat{i} - \cos(t)\hat{j}$, and $\kappa(t) = \cos(t)$.

2. N/A

3. (a) $\kappa(x) = \frac{e^x}{(1+e^{2x})^{3/2}}$

   (b) $\kappa(x) = \frac{|\sin(x)|}{(1+\cos^2(x))^{3/2}}$

4. $\kappa = \frac{2}{3\sqrt{3}}$

5. (a) 2.

   (b) $\overrightarrow{N}(1) = \langle -1, 0 \rangle$.

   (c) $(-4, 0)$.

   (d) $(x+4)^2 + y^2 = 4$.

Math 2550 Worksheet Section 13.5

1. Write $\vec{a}$ in the form of $\vec{a} = a_T\vec{T} + a_N\vec{N}$ without finding $\vec{T}$ and $\vec{N}$ for $\vec{r}(t) = \langle a\sin t, a\cos t, bt\rangle$.

2. Find $\vec{T}$, $\vec{N}$, $\vec{B}$, κ, and τ for

   (a) $\vec{r}(t) = (3\sin(2t))\hat{i} - (3\cos(2t))\hat{j} + 2t\hat{k}$.

   (b) $\vec{r}(t) = (a\sin t)\hat{i} + (a\cos t)\hat{j} + bt\hat{k}$.

3. Find the equations for the osculating, normal, and rectifying planes at the given value of t.

   (a) $\vec{r}(t) = (e^t\cos(t))\hat{i} + (e^t\sin(t))\hat{j} + 2\hat{k}$, $t = 0$.

   (b) $\vec{r}(t) = t^2\hat{i} + (t^3 - 1)\hat{j} + e^t\hat{k}$, $t = 0$.

Answer Key

1. $\vec{a} = |a|\vec{N}$.

2. (a)

$$\vec{T} = \frac{\langle 3\cos 2t, 3\sin 2t, 1\rangle}{\sqrt{10}}$$

$$\vec{N} = \frac{\vec{T}'}{|T'|} = \langle -\sin 2t, \cos 2t, 0\rangle$$

$$\vec{B} = \frac{\langle -\cos 2t, \sin 2t, 3\rangle}{\sqrt{10}}$$

$$\kappa = \frac{3}{10}$$

$$\tau = \frac{1}{10}$$

(b)

$$\vec{T} = \frac{\langle a\cos t, -a\sin t, b\rangle}{\sqrt{a^2+b^2}}$$

$$\vec{N} = \langle -\sin t, -\cos t, 0\rangle$$

$$\vec{B} = \frac{\langle b\cos t, -b\sin t, -a\rangle}{\sqrt{a^2+b^2}}.$$

$$\kappa = \frac{a}{a^2+b^2}$$

$$\tau = \frac{-b}{a^2+b^2}$$

3. (a) Osculating Plane: $z = 2$, Normal Plane: $x + y = 1$ and Rectifying Plane: $-x + y = -1$.

   (b) Osculating Plane: $y = -1$, Normal Plane: $z = 1$ and Rectifying Plane: $x = 0$.

Math 2550 Worksheet Section 14.1

1. Find and sketch the domain for each function.
   (a) $f(x, y) = \sqrt{x - y - 1}$.
   (b) $f(x, y) = \sqrt{(x - 4)(y^2 - 1)}$.
   (c) $f(x, y) = \cos^{-1}(y - 4x^2)$.
   (d) $f(x, y) = \dfrac{1}{4 - x^2 - y^2}$.
   (e) $f(x, y) = \dfrac{1}{\ln(4 - x^2 - y^2)}$
2. Let $f(x, y) = \sqrt{1 - xy}$.
   (a) Find and sketch the domain of f.
   (b) Sketch the level curve $f(x, y) = 2$.
3. Find an equation for the level curve of the function $F(x, y) = \dfrac{2y - x}{x + y + 1}$ passing through $(-1, 1)$.
4. Find the equation for the level surface of the function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ passing through $(1, 1, 1)$.

Answer Key

1. (a) Domain: $\{(x,y) \mid x-y-1 \geq 0\}$
   (b) Domain: $\{(x,y) \mid (x-4)(y^2-1) \geq 0\}$
   (c) Domain: $\{(x,y) \mid 4x^2-1 \leq y \leq 4x^2+1\}$
   (d) Domain: $\{(x,y) \mid x^2+y^2 \neq 4\}$
   (e) Domain: $\{(x,y) \mid x^2+y^2 < 4, \quad x^2+y^2 \neq 3\}$
2. (a) Domain: $\{(x,y) \mid xy \leq 1\}$

   (b) $\sqrt{1-xy} = 2 \implies y = -\frac{3}{x}$.

3. $y = -4x-3, \quad (x,y) \neq (-\frac{2}{3}, -\frac{1}{3})$.
4. $x^2+y^2+z^2 = 3$

Math 2550 Worksheet Section 14.2

1. Let $f(x,y) = \dfrac{x-2y}{x^3-8y^3}$. Find $\displaystyle\lim_{(x,y)\to(2,1)} f(x,y)$ or show it does not exist.

2. Let $f(x,y) = \dfrac{\sqrt{2x-y}-2}{2x-y-4}$. Find $\displaystyle\lim_{(x,y)\to(2,0)} f(x,y)$ or show it does not exist.

3. At what points (x,y) in the plane is $f(x,y) = \cos\left(\dfrac{1}{xy}\right)$ continuous?

4. At what points (x,y,z) is $h(x,y,z) = \dfrac{1}{1-\ln\left(x^2+y^2+z^2\right)}$ continuous?

Answer Key

1. $\frac{1}{12}$.
2. $\frac{1}{4}$.
3. $\{(x, y)|\, x \neq 0, y \neq 0\}$
4. $\{(x, y)|\, x^2 + y^2 + z^2 > 0, x^2 + y^2 + z^2 \neq e\}$

Math 2550 Worksheet Section 14.3

1. Find f_x and f_y for:

   (a) $f(x,y) = \left(xy + \frac{y}{3}\right)^{3/2}$

   (b) $f(x,y) = e^{x^2 y} \ln x$

   (c) $f(x,y) = \sum_{n=0}^{\infty} (xy)^n \qquad (|xy| < 1)$

2. Find f_x, f_y, and f_z for the function $f(x,y,z) = z^{x^y} \qquad (x > 0,\ y > 0,\ z > 0)$.

3. Let $f(x,y) = x^2 y^2$. Find $f_y(a,b)$ using the limit definition of the partial derivative.

4. Find all the second partial derivatives for $f(x,y) = e^x + x \ln y$.

Answer Key

1. (a) $f_x = \frac{3}{2}y(xy + \frac{y}{3})^{1/2},\ f_y = \frac{3}{2}(xy + \frac{y}{3})^{1/2}(x + \frac{1}{3})$

   (b) $f_x = 2xye^{x^2y}\ln x + \frac{e^{x^2y}}{x},\ f_y = x^2e^{x^2y}\ln x$

   (c) $f_x = \frac{y}{(1-xy)^2},\ f_y = \frac{x}{(1-xy)^2}$

2. $f_x = x^{y-1}yz^{x^y}\ln z,\ f_y = x^y\ln x \cdot z^{x^y}\ln z,\ f_z = x^yz^{x^y-1}$.

3. $2a^2b$.

4. $f_x = e^x + \ln(y),\ f_y = \frac{x}{y}$, and so $f_{xx} = e^x,\ f_{xy} = \frac{1}{y}, f_{yy} = -\frac{x}{y^2}$

Math 2550 Worksheet Section 14.4

1. Find $\dfrac{dw}{dt}$ when $t = 1$, if $w = 2ye^x - \ln z$, $\;x = \ln(t^2+1)$, $\;y = \tan^{-1}(t)$, $\;z = e^t$.

2. Let $w = xy + yz + zx$, where $x = r\cos\theta$, $\;y = r\sin\theta$, $\;z = r\theta$.

   Find $\dfrac{\partial w}{\partial r}$ and $\dfrac{\partial w}{\partial \theta}$ when $r = 2$ and $\theta = \dfrac{\pi}{2}$.

3. Find $\dfrac{dy}{dx}$ if $\tan^{-1}(x^2y) = x + xy^2$.

4. Suppose that we subsitute polar coordinates $x = r\cos\theta$ and $y = r\sin\theta$ in a differentiable function $w = f(x,y)$.

   (a) Show that

   $$\frac{\partial w}{\partial r} = f_x\cos\theta + f_y\sin\theta \qquad \text{and} \qquad \frac{1}{r}\frac{\partial w}{\partial \theta} = -f_x\sin\theta + f_y\cos\theta$$

   (b) Solve the equations in part (a) to express f_x and f_y in terms of $\dfrac{\partial w}{\partial r}$ and $\dfrac{\partial w}{\partial \theta}$.

Answer Key

1. $\pi + 1$.
2. $\frac{\partial \omega}{\partial r} = 2\pi$ and $\frac{\partial \omega}{\partial \theta} = -2\pi$
3. $\frac{x^4y^4+x^4y^2+y^2-2xy+1}{-2x^5y^3+x^2-2xy}$
4. You can do this by using chain rules.

Math 2550 Worksheet Section 14.5

1. Let
$$f(x,y) = x^2y + e^{-x} + e^{xy}\sin y, \qquad P = (1,0).$$
   (a) Find the directions in which the function f increases and decreases most rapidly at P.
   (b) Find all the unit vectors $\vec{u}$ such that the directional derivative $D_{\mathbf{u}}f(P) = 0$.
2. Let $f(x,y) = xy$. Sketch the curve $f(x,y) = -4$ together with ∇f and the tangent line at $(2,-2)$. Then, find write an equation for the tangent line.
3. The directional derivative of some differentiable function $f(x,y)$ at $(2,1)$ in the direction going from $(2,1)$ toward the point $(1,3)$ is $-\frac{2}{\sqrt{5}}$, and the directional derivative of f at $(2,1)$ in the direction going from $(2,1)$ toward the point $(5,5)$ is 1. Compute $f_x(2,1)$ and $f_y(2,1)$.

Answer Key

1. (a) The directions in which f increases most rapidly is $\langle \frac{-1}{\sqrt{4e^2+1}}, \frac{2e}{\sqrt{4e^2+1}} \rangle$. The directions in which f decreases most rapidly is $\langle \frac{1}{\sqrt{4e^2+1}}, \frac{-2e}{\sqrt{4e^2+1}} \rangle$.

   (b) The vectors are $\langle \frac{2e}{\sqrt{4e^2+1}}, \frac{1}{\sqrt{4e^2+1}} \rangle$ or $\langle \frac{-2e}{\sqrt{4e^2+1}}, \frac{-1}{\sqrt{4e^2+1}} \rangle$

2. $y = x - 4$.

3. $f_x(2, 1) = 1.8, f_y(2, 1) = -0.1$

Math 2550 Worksheet Section 14.6

1. Find the equations for the tangent plane and normal line at the point $P_0(0,1,2)$ of the surface
$$\cos(\pi x) - x^2 y + e^{xz} + z = 4.$$

2. Let
$$f(x,y,z) = e^x \cos(yz).$$
Estimate the change df in f, where we move $ds = 0.1$ in the direction $\vec{v} = 2\hat{i} + 2\hat{j} - 2\hat{k}$ from a general point $P_0(x,y,z)$ and in particular at $(0,0,0)$.

3. Find parametric equations for the line tangent to the curve of intersection of the surfaces
$$xyz = 1 \quad \text{and} \quad x^2 + 2y^2 + 3z^2 = 6$$
at the point $(1,1,1)$.

4. Find the linearization of $f(x,y,z) = \tan^{-1}(xyz)$ at $(1,1,0)$.

5. Find the linearization of the function $f(x,y) = 1 + y + x\cos y$ at $P_0(0,0)$ and find an upper bound for the magnitude $|E|$ of the error in the approximation over the rectangle $R : |x| \le 0.2,\ |y| \le 0.2$.

Answer Key

1. Tangent plane is $2x + z = 2$ and normal line is $x = 2t$, $y = 1$, $z = t + 2$.
2. $\frac{1}{10\sqrt{3}}$
3. $x = 1 + 2t, y = 1 - 4t, z = 1 + 2t$
4. $L = z$
5. $L = 1 + x + y$ and $E \leq 0.016$.

Math 2550 Worksheet Section 14.7

1. Find all the local maxima, local minima, and saddle points of $f(x, y) = e^y(x^2 - y^2)$.
2. Find the absolute maxima and minima of the function $f(x, y) = x^2 - xy + y^2 + 1$ on the closed triangular plate bounded by lines $x = 0$, $y = 4$, $y = x$ in the first quadrant.
3. Among all rectangular boxes of volume 27 cm^3, what are the dimensions of the box with the smallest surface area? What is the smallest possible surface area? (assume this occurs at a local min of the surface area function)
4. In each case, the origin is a critical point of f and $f_{xx}f_{yy} - (f_{xy})^2 = 0$ at the origin, so the Second Derivative Test fails at the origin. Use some other method to determine whether the function f has a maximum, a minimum, or neither at the origin.
   (a) $f(x, y) = x^2y^2$
   (b) $f(x, y) = 1 - x^2y^2$
   (c) $f(x, y) = xy^2$
   (d) $f(x, y) = x^3y^2$
   (e) $f(x, y) = x^3y^3$
   (f) $f(x, y) = x^4y^4$

Answer Key

1. $(0,0)$ is a saddle point and $(0,-2)$ is a minimum.
2. Absolute maximum of 17 at $(0,4)$ and $(4,4)$, Absolute minimum of 1 at $(0,0)$.
3. $3 \times 3 \times 3$ and 54.
4. (a) Minimum is 0 at $(0,0)$.
   (b) Maximum is 1 at $(0,0)$.
   (c) Neither.
   (d) Neither.
   (e) Neither.
   (f) Minimum is 0 at $(0,0)$.

Math 2550 Worksheet Section 14.8

1. Find the maximum and minimum values of x^2y subject to the constraint $x^2 + 2y^2 = 6$.
2. Find the point on the plane $x + 2y + 3z = 13$ closest to the point $(1, 1, 1)$.
3. Find the maximum value that $f(x, y, z) = x^2 + 2y - z^2$ can have on the line of intersection of the planes $2x - y = 0$ and $y + z = 0$.
4. Find the maximum and minimum values of $f(x, y, z) = x - 2y + 5z$ on the sphere $x^2 + y^2 + z^2 = 14$.

Answer Key

1. Maximum of 4 and minimum of -4.
2. $\left(\frac{3}{2}, 2, \frac{5}{2}\right)$.
3. Maximum of $\frac{4}{3}$.
4. Maximum of $2\sqrt{105}$ and minimum of $-2\sqrt{105}$.

Math 2550 Worksheet Section 15.1 and 15.2

1. Find $\displaystyle\iint_R \frac{xy^2}{x^2+1}\,dA,\qquad R\colon\ 0 \le x \le 1,\ \ -3 \le y \le 3$

2. Write an iterated integral for $\iint_R dA$ over the region R using vertical cross-sections and horizontal cross-sections.

   (a) Bounded by $y = e^{-x}$, $y = 1$, and $x = \ln 3$.

   (b) Bounded by $y = x^2$ and $y = x + 2$

3. Sketch the region of integration, reverse the order of integration, and evaluate the integral.

   (a) $\displaystyle\int_0^{\sqrt{\pi}} \int_y^{\sqrt{\pi}} \cos(x^2)\,dx\,dy.$

   (b) $\displaystyle\int_0^{8} \int_{\sqrt[3]{x}}^{2} e^{y^4}\,dy\,dx.$

4. Find the volume of the solid bounded by the cylinder $y^2 + z^2 = 4$ and the planes $x = 2y$, $x = 0$, $z = 0$ in the first octant.

Answer Key

1. $9 \ln 2$.
2. (a) Using vertical cross-section, we get
$$\int_0^{\ln 3} \int_{e^{-x}}^{1} dy\ dx.$$
Using horizontal cross-section, we get
$$\int_{1/3}^{1} \int_{-\ln y}^{\ln 3} dx\ dy.$$
(b) Using vertical cross-section, we get
$$\int_{-1}^{2} \int_{x^2}^{x+2} dy\ dx.$$
Using horizontal cross-section, we get
$$\int_0^1 \int_{-\sqrt{y}}^{\sqrt{y}} dx\ dy + \int_1^4 \int_{y-2}^{\sqrt{y}} dx\ dy.$$
3. (a) 0.

   (b) $\frac{1}{4}(e^{16} - 1)$.
4. $\frac{16}{3}$.

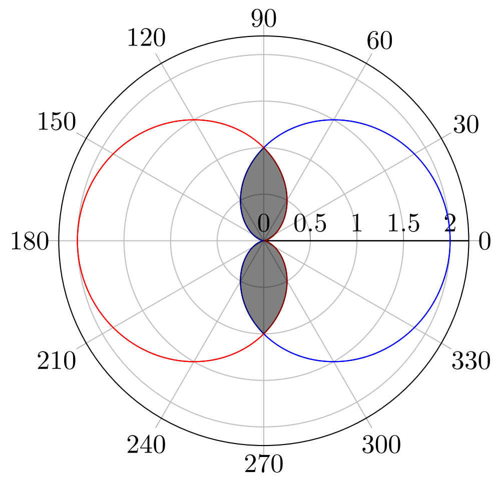
Math 2550 Worksheet Section 15.3 and 15.4

1. Sketch the region bounded by
$$y = 1 - x, \quad y = 2, \quad y = e^x$$
and find the area of of the region.

2. Change the Cartesian integral
$$\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} e^{-x^2-y^2}\, dx\, dy$$
into an equivalent polar integral and evaluate the integral.

3. Use polar coordinates to find the volume of the solid above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 1$.

4. Find the area of the region common to the interiors of the cardioids $r = 1 + \cos\theta$ and $r = 1 - \cos\theta$.



5. Let E be the part of $x^2 + y^2 + z^2 = 4$ when $z \geq 0$ and $y \geq 1$. Find the volume of E via polar integral.

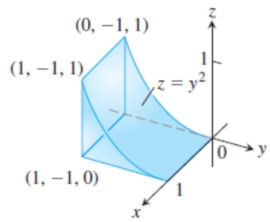
Answer Key

1. $2\ln 2 - \dfrac{1}{2}$.
2. $\dfrac{\pi}{2}(1 - e^{-4})$.
3. $\dfrac{\pi}{3}(2 - \sqrt{2})$.
4. $\dfrac{3\pi}{2} - 4$.
5. $\displaystyle\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \int_{\frac{1}{\sin\theta}}^{2} \sqrt{4 - r^2}\ r\ dr\ d\theta$

Math 2550 Worksheet Section 15.5

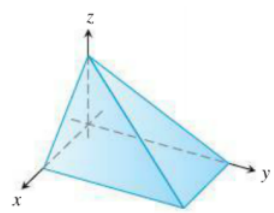
1. Evaluate $\iiint_E z dV$, where E is the solid tetrahedron bounded by the four planes $x = 0$, $y = 0$, $z = 0$, and $x + y + z = 1$. Include a sketch of the solid.

2. Set up integrals that would calculate the volume of the region below, using the specified orders of integration.



(a) $dy\ dz\ dx$ (b) $dy\ dx\ dz$ (c) $dx\ dy\ dz$ (d) $dx\ dz\ dy$ (e) $dz\ dx\ dy$

3. Find the volume of the region in the first octant bounded by the coordinate planes and the planes $x + z = 1$ and $y + 2z = 2$.



4. Evaluate the integral

$$\int_0^{\pi/2} \int_0^y \int_0^x \cos(x + y + z)\ dz\ dx\ dy$$

Answer Key

1. $\frac{1}{24}$.
2. (a) $\int_0^1 \int_0^1 \int_{-1}^{-\sqrt{z}} \; dy \; dz \; dx$
   (b) $\int_0^1 \int_0^1 \int_{-1}^{-\sqrt{z}} \; dy \; dx \; dz$
   (c) $\int_0^1 \int_{-1}^{-\sqrt{z}} \int_0^1 \; dx \; dy \; dz$
   (d) $\int_{-1}^0 \int_0^{y^2} \int_0^1 \; dx \; dz \; dy$
   (e) $\int_{-1}^0 \int_0^1 \int_0^{y^2} \; dz \; dx \; dy$
3. $\frac{2}{3}$.
4. $-\frac{1}{3}$.

Math 2550 Worksheet Section 15.7

1. Convert the integral $\int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}}\int_{0}^{x}(x^2+y^2)\,dz\,dx\,dy$ into an integral in cylindrical coordinates, and evaluate the integral.

2. Let D be the right circular cylinder whose base is the circle $r = 2\sin\theta$ in the xy-plane and whose top lies in plane $z = 4 - y$. Recall that $r = 2\sin\theta$ describes a circle centered at $(0, 1)$ with radius 1 in the xy-plane. Using cylindrical coordinates,

   (a) find the volume of the region D.

   (b) find the $\bar{x}$ component of the centroid of the region (hint: use symmetry).

3. Find the volume of the solid that is between the spheres $\rho = \sqrt{2}$ and $\rho = 2$, but outside of the circular cylinder $x^2 + y^2 = 1$.

4. Suppose $a \geq 0$. Find the volume of the region cut from the solid sphere $\rho \leq a$ by the half-planes $\theta = 0$ and $\theta = \pi/6$ in the first octant.

Answer Key

1. $\dfrac{2}{5}$.
2. (a) 3π.

   (b) $\bar{x} = 0$.
3. $\dfrac{12\sqrt{3} - 4}{3}\pi$.
4. $\dfrac{a^3\pi}{18}$.

Math 2550 Worksheet Section 15.8

1. Let R be the parallelogram in the first quadrant bounded by the lines

$$y = -2x + 4; \quad y = -2x + 7; \quad y = x - 2; \quad y = x + 1.$$

   Evaluate

$$I = \int\int_R 2x^2 - xy - y^2 \, dx \, dy$$

   by using the transformation $u = x - y$ and $v = 2x + y$ in the following steps:

   (a) Sketch R.

   (b) Solve for x, y in terms of u, v.

   (c) Describe the region in the uv-plane that corresponds to R.

   (d) Find the Jacobian $\dfrac{\partial(x, y)}{\partial(u, v)}$.

   (e) Use the substitution rule for double integrals to evaluate I.

2. Use the transformation $x = u^2 - v^2$ and $y = 2uv$ to evaluate the integral

$$\int_0^1 \int_0^{2\sqrt{1-x}} \sqrt{x^2 + y^2} \, dy \, dx.$$

3. Let R be the region in the first quadrant of the xy-plane bounded by

$$xy = 4; \quad xy = 16; \quad y = x; \quad y = 4x.$$

   Use the transformation $x = u/v$ and $y = uv$ with $u > 0$ and $v > 0$ to evaluate

$$\int\int_R \left(\frac{y}{x}\right)^2 + \frac{1}{xy} \, dx \, dy.$$

Answer Key

1. (e) $\frac{33}{4}$.
2. $\frac{56}{45}$.
3. $45 + 2(\ln 2)^2$.