

Taker Name:

Key

GTID: 90

Section:

Grader #1:

GTID: 90

§§13.3: Arc-length parameter

Find the length of the the curve segment

$$\mathbf{r}(t) = \langle t \sin t + \cos t, t \cos t - \sin t, 0 \rangle, \quad 1 \leq t \leq 3.$$

$$\text{Hint: } L = \int_a^b \|\mathbf{r}'(t)\| dt.$$

$$\begin{aligned} \mathbf{r}'(t) &= \langle \cancel{\sin t} - t \cancel{\cos t} - \cancel{\sin t}, \cancel{\cos t} - t \cancel{\sin t} - \cancel{\cos t} \rangle \\ &= \langle -t \cos t, -t \sin t \rangle \end{aligned}$$

$$\|\mathbf{r}'(t)\|^2 = t^2 \cos^2 t + t^2 \sin^2 t = t^2 \Rightarrow \|\mathbf{r}'(t)\| = |t| = t \quad (\text{since } t \geq 0)$$

$$\text{So } L = \int_1^3 t \, dt = \frac{1}{2} t^2 \Big|_1^3$$

$$= \frac{9}{2} - \frac{1}{2} = \frac{8}{2} = 4$$

A

J

N

G2:

A

J

N

G3:

A

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N