

Taker Name:

Key

GTID: 90

Section:

Grader #1:

GTID: 90

§§13.3: Arc-length parameter

Find the arc-length parameter $s(t)$ for the curve

$$\mathbf{r}(t) = \langle t \sin t + \cos t, -t \cos t + \sin t \rangle, \quad 0 \leq t \leq \pi.$$

$$\text{Hint: } s(t) = \int_{t_0}^t \|\mathbf{r}'(\tau)\| d\tau.$$

$$\begin{aligned} \vec{r}'(t) &= \langle \cancel{\sin t} - t \cancel{\cos t} - \cancel{\sin t}, -\cancel{\cos t} + t \cancel{\sin t} + \cancel{\cos t} \rangle \\ &= \langle -t \cos t, t \sin t \rangle \end{aligned}$$

$$\|\vec{r}'(t)\|^2 = t^2 \cos^2 t + t^2 \sin^2 t = t^2 \Rightarrow \|\vec{r}'(t)\| = |t| = t \quad (\text{since } t \geq 0)$$

$$\begin{aligned} \text{So } s(t) &= \int_0^t \tau d\tau = \frac{1}{2} \tau^2 \Big|_0^t \\ &= \boxed{\frac{1}{2} t^2} \end{aligned}$$

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G2:

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