Taker Name: Key GTID: 903 Section:

Grader #1: GTID: 903

§14.8: Constrained optimization

Find the closest points on the curve to the origin.

$$xy^2 = 54$$

$54 = 3^3 * 2 = 9*6$

$27 \quad 2$

$d(x,y) = \sqrt{x^2+y^2}$ distance of (x,y) to $(0,0)$.

minimize $f(x,y) = x^2+y^2$ instead.

Subject to constraint $g(x,y) = xy^2 = 54$.

$\nabla f = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$ and $\nabla g = \begin{bmatrix} y^2 \\ 2xy \end{bmatrix}$.

Solve $\begin{cases} \nabla f = \lambda \nabla g \\ g = k \end{cases}$ $\begin{cases} ① \ 2x = \lambda y^2 \\ ② \ 2y = \lambda 2xy \\ ③ \ xy^2 = 54 \end{cases}$

From ② $y=0$ or $\lambda x = 1$

Case 1: $y=0$. This is impossible since by constraint ④ $xy^2 = 54$, so $x \neq 0$ and $y \neq 0$.

Case 2: $\lambda x = 1$ Then $x \neq 0$ and $\lambda = 1/x$, From ① $2x = \frac{1}{x}y^2$

$\Rightarrow 2x^2 = y^2$

Sub into ④ get $2x^3 = 54 \Rightarrow x^3 = 27 \Rightarrow x = 3$

So $x = 3$ and $y^2 = 54/3 = 18$ So $y = \pm\sqrt{18} = \pm 3\sqrt{2}$.

closest points are $(3, 3\sqrt{2})$ & $(3, -3\sqrt{2})$

A	
J	
N	
A	
J	
N	
A	
J	
N	