Taker Name: Key GTID: 903 Section:

Grader #1: GTID: 903

§14.8: Constrained optimization

Find the minimum distance for points on surface to the origin.

$$z^2 = x^2 - y + 3$$

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$d(x,y,z) = \sqrt{x^2+y^2+z^2}$ distance of (x,y,z) to $(0,0,0)$.

minimize $f(x,y,z) = x^2+y^2+z^2$ instead.

Subject to constraint $g(x,y,z) = z^2 - x^2 + y = 3$ $(k=3)$

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$\nabla f = \begin{bmatrix} 2x \\ 2y \\ 2z \end{bmatrix}$ and $\nabla g = \begin{bmatrix} -2x \\ 1 \\ 2z \end{bmatrix}$.

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Solve $\begin{cases} \nabla f = \lambda \nabla g \\ g = k \end{cases}$ $\begin{cases} ① \ 2x = -\lambda 2x \\ ② \ 2y = \lambda \\ ③ \ 2z = \lambda 2z \\ ④ \ z^2 - x^2 + y = 3 \end{cases}$

From ③ $z=0$ or $\lambda = 1$.

Case 1: $\lambda = 1$. Then $2x = -2x \Rightarrow x = 0$

$2y = 1 \Rightarrow y = 1/2$

So ④ becomes $z^2 + \frac{1}{2} = 3 \Rightarrow z = \sqrt{5/2}$

get $(0, 1/2, \sqrt{5/2})$

Case 2: $z=0$. From ① $x=0$ or $\lambda = -1$. If $x=0$ then $y=3$

get $(0,3,0)$.

If $\lambda = -1$, then $y = 1/2$ and ④ $0 - x^2 + \frac{1}{2} = 3 \Rightarrow x^2 = -2.5$ impossible!

So closer $d(0,3,0) = 3$ $d(0, \frac{1}{2}, \sqrt{5/2}) = \sqrt{\frac{1}{4} + \frac{5}{2}} = \sqrt{11/4} = \frac{\sqrt{11}}{2}$ MIN DISTANCE