Taker Name: Key GTID: 903 Section:

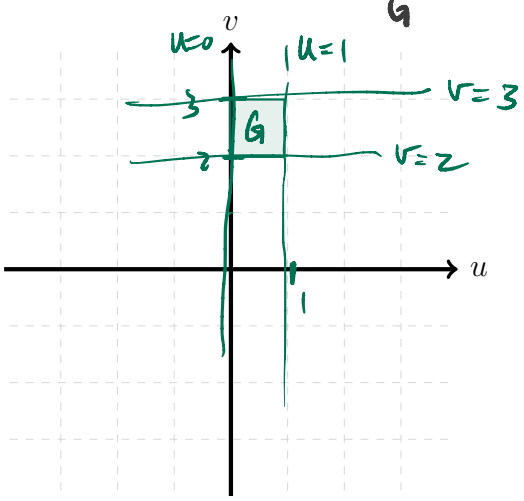
Grader #1: GTID: 903

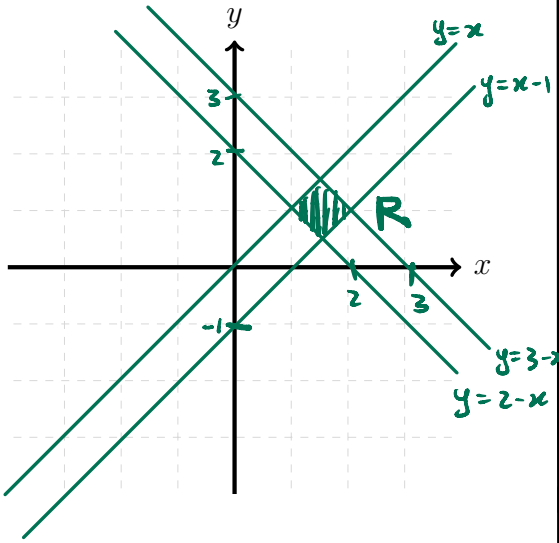
§15.5: Triple integrals

Let $u = x - y$ and $v = x + y$. Set up *but do* NOT *evaluate*

$$M = \iint_R x + y \, dA$$

where R is the region in the first quadrant bounded by the lines $x - y = 0$, $x - y = 1$, $x + y = 2$, $x + y = 3$. On the axes below, (a) sketch the new region of integration G after the change of variables and the original region R, (b) find the transformation $\mathbf{T}(u, v) = (x, y)$ and (c) compute the Jacobian determinant $|\det \mathbf{T}(u, v)|$. Finally, (d) write the new iterated integral after the change of coordinates $\iint_G f(\mathbf{T}(u, v)) \, |\det \mathbf{T}(u, v)| \, du \, dv$.





A		
J		
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(a) $x - y = u = 0$ to $x - y = u = 1$ So $u \in [0, 1]$

and $x + y = v = 2$ to $x + y = v = 3$ So $v \in [2, 3]$

(d) $M = \int_0^1 \int_2^3 v \, dv \, du$

(b) $x - y = u$, $x + y = v$ $\Rightarrow$ $2x = u + v$, $2y = u - v$ $\Rightarrow$ $x = \frac{u+v}{2}$, $y = \frac{u-v}{2}$

$$T\left(\begin{bmatrix} u \\ v \end{bmatrix}\right) = \begin{bmatrix} (u+v)/2 \\ (u-v)/2 \end{bmatrix}$$

(c) $|\det DT| = \begin{vmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{vmatrix} = \left| -\frac{1}{4} - \frac{1}{4} \right| = |-1/2| = 1/2$