MATH 2550 G/J Midterm 1
VERSION A
Fall 2025
COVERS SECTIONS 12.1-12.6, 13.1-13.4, 14.1-14.2

Full name: ______________________________ **GT ID:**____________

Honor code statement: I will abide strictly by the Georgia Tech honor code at all times. I will not use a calculator. **I do not have a phone within reach**, and I will not reference any website, application, or other CAS-enabled service. I will not consult with my notes or anyone during this exam. I will not provide aid to anyone else during this exam.

(   ) All of the knowledge presented in this exam is entirely my own. I am initialing to the left to attest to my integrity.

Read all instructions carefully before beginning.

- Print your name and GT ID neatly above.
- You have 50 minutes to take the exam.
- You may not use aids of any kind.
- Please show your work [J] and annotate your work using proper notation [N].
- Good luck!

Question	Points
1	2
2	10
3	12
4	10
5	8
6	8
Total:	50

For T/F problems choose whether the statement is true or false. If the statement is *always* true, pick true. If the statement is *ever* false, pick false. Also please be sure to neatly fill in the bubble corresponding to your answer choice. [A]

1. (2 points) If $\mathbf{u}$ and $\mathbf{v}$ are vectors in $\mathbb{R}^3$, then $\mathbf{u} \times \mathbf{v} = \mathbf{v} \times \mathbf{u}$.

   ◯ **TRUE** ◯ **FALSE**

2. (10 points) Find the equation of the plane passing through the point $P(1, 1, 0)$ which contains the lines $\ell_1(t) = \langle 1, 1, 0\rangle + t\langle 1, 3, -1\rangle$ and $\ell_2(s) = \langle 1, 2, -1\rangle + s\langle 0, 1, -1\rangle$, $s, t \in \mathbb{R}$.
[AJN]

3. (12 points) Let $\mathbf{r}(t) = \langle \cos^3 t, \sin^3 t \rangle$, $0 \leq t \leq \pi/2$. Find the curve's unit tangent vector $\mathbf{T}(t)$ and the length of the curve parametrized by $\mathbf{r}(t)$. [AJN]

$\mathbf{T}(t) =$ [ ]

length is [ ]

4. (10 points) In this problem, you will work with the helix curve

$$\mathbf{r}(t) = \sin(3t)\mathbf{i} + \cos(3t)\mathbf{j} + t\mathbf{k}$$

for $0 \leq t \leq 2\pi$. [AJN]

(a) Compute the principal unit normal vector $\mathbf{N}(t)$.

(b) Compute the curvature $\kappa(t)$.

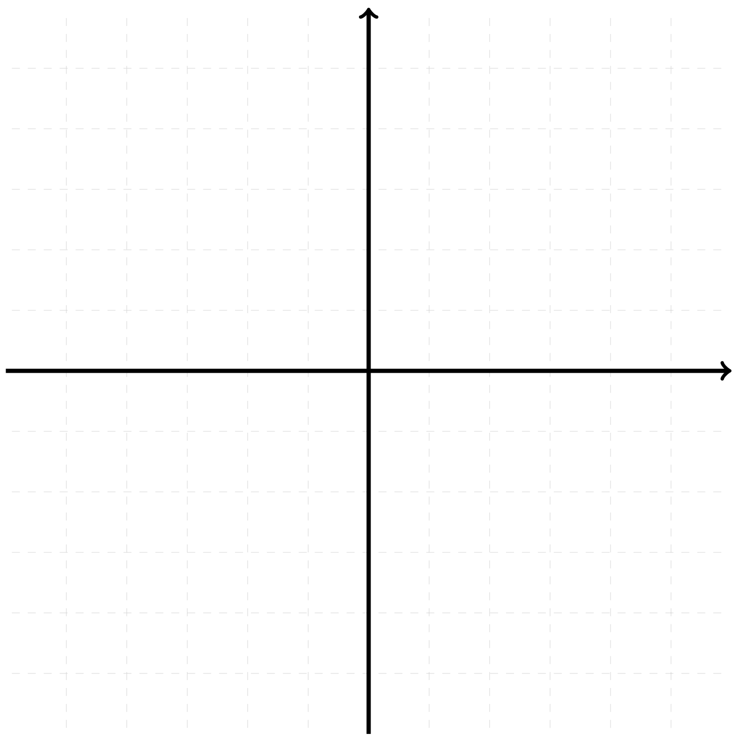
$\mathbf{N}(t) =$ [ ]

$\kappa(t) =$ [ ]

5. (8 points) Draw a contour map on the axes provided including all three of the level curves $g(x, y) = c$ for the function

$$g(x, y) = 20 - \frac{1}{2}x^2 - \frac{1}{2}y^2, \quad c = 2, 12, 18.$$

Show your work for how you find the equation of each level set, include labels for the axes, and label each level set as well as an x-intercept and y-intercept of each level set. [AJN]



6. (8 points) Show that the limit does not exist. To receive full credit, you must show work supporting your answer, use proper limit notation, and mention the test that you are using. [AJN]

$$\lim_{(x,y)\to(0,0)} \frac{xy}{|xy|}$$

FORMULA SHEET

- $\langle u_1, u_2, u_3\rangle \cdot \langle v_1, v_2, v_3\rangle = u_1v_1 + u_2v_2 + u_3v_3$
- $\mathbf{u}\cdot\mathbf{v} = |\mathbf{u}||\mathbf{v}|\cos(\theta)$
- $\langle u_1, u_2, u_3\rangle \times \langle v_1, v_2, v_3\rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$
- $|\mathbf{u}\times\mathbf{v}| = |\mathbf{u}||\mathbf{v}||\sin(\theta)|$
- $L = \displaystyle\int_a^b |\mathbf{r}'(t)|\ dt$
- $s(t) = \displaystyle\int_{t_0}^t |\mathbf{r}'(\tau)|\ d\tau$
- $\mathbf{T} = \dfrac{\mathbf{v}}{|\mathbf{v}|} = \dfrac{d\mathbf{r}}{ds}$
- $\kappa = \left|\dfrac{d\mathbf{T}}{ds}\right| = \dfrac{1}{|\mathbf{v}|}\left|\dfrac{d\mathbf{T}}{dt}\right| = \dfrac{|\mathbf{v}\times\mathbf{a}|}{|\mathbf{v}|^3}$
- $\mathbf{N} = \dfrac{1}{\kappa}\dfrac{d\mathbf{T}}{ds} = \dfrac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|}$

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