MATH 2550 G/J Midterm 1
VERSION A
Fall 2025
COVERS SECTIONS 12.1-12.6, 13.1-13.4, 14.1-14.2

Full name: Key **GT ID:**

Honor code statement: I will abide strictly by the Georgia Tech honor code at all times. I will not use a calculator. **I do not have a phone within reach**, and I will not reference any website, application, or other CAS-enabled service. I will not consult with my notes or anyone during this exam. I will not provide aid to anyone else during this exam.

(   ) All of the knowledge presented in this exam is entirely my own. I am initialing to the left to attest to my integrity.

Read all instructions carefully before beginning.

- Print your name and GT ID neatly above.
- You have 50 minutes to take the exam.
- You may not use aids of any kind.
- Please show your work [J] and annotate your work using proper notation [N].
- Good luck!

Question	Points
1	2
2	10
3	12
4	10
5	8
6	8
Total:	50

For T/F problems choose whether the statement is true or false. If the statement is *always* true, pick true. If the statement is *ever* false, pick false. Also please be sure to neatly fill in the bubble corresponding to your answer choice. [A]

1. (2 points) If $\mathbf{u}$ and $\mathbf{v}$ are vectors in $\mathbb{R}^3$, then $\mathbf{u} \times \mathbf{v} = \mathbf{v} \times \mathbf{u}$.

   ○ **TRUE** ● **FALSE**

2. (10 points) Find the equation of the plane passing through the point $P(1, 1, 0)$ which contains the lines $\ell_1(t) = \langle 1, 1, 0\rangle + t\langle 1, 3, -1\rangle$ and $\ell_2(s) = \langle 1, 2, -1\rangle + s\langle 0, 1, -1\rangle$, $s, t \in \mathbb{R}$. [AJN]

$$\boxed{-2x + y + z = -1}$$

$v_1 = \langle 1, 3, -1\rangle$

$v_2 = \langle 0, 1, -1\rangle$

$$n = v_1 \times v_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & -1 \\ 0 & 1 & -1 \end{vmatrix} = \hat{i}(-3+1) - \hat{j}(-1) + \hat{k}(1)$$

$$= \langle -2, 1, 1\rangle$$

So plane is $-2x + y + z = d$

& point $P(1,1,0)$ so

$$-2 + 1 + 0 = d \Rightarrow d = -1$$

Plane is $-2x + y + z = -1$

3. (12 points) Let $\mathbf{r}(t) = \langle \cos^3 t, \sin^3 t \rangle$, $0 \leq t \leq \pi/2$. Find the curve's unit tangent vector $\mathbf{T}(t)$ and the length of the curve parametrized by $\mathbf{r}(t)$. [AJN]

$\mathbf{T}(t) =$ $\boxed{\langle -\cos t, \sin t \rangle}$

length is $\boxed{3/2}$

$$r'(t) = \langle -3\cos^2 t \sin t, 3\sin^2 t \cos t \rangle$$

$$\|r'(t)\|^2 = 9\cos^4 t \sin^2 t + 9\sin^4 t \cos^2 t$$

$$= 9\cos^2 t \sin^2 t (\cos^2 t + \sin^2 t)$$

$$= 9\cos^2 t \sin^2 t$$

so $\|r(t)\| = 3\cos t \sin t$

$$T(t) = \frac{r'(t)}{\|r'(t)\|} = \langle -\cos t, \sin t \rangle$$

and

$$L = \int_0^{\pi/2} 3\cos t \sin t \, dt = \int_0^1 3u \, du$$

$\boxed{u = \sin t \quad du = \cos t \, dt}$

$t = 0 \quad u = 0$
$t = \pi/2 \quad u = 1$

$$= \frac{3}{2}u^2 \Big|_0^1 = \frac{3}{2} - 0 = \frac{3}{2}$$

4. (10 points) In this problem, you will work with the helix curve

$$\mathbf{r}(t) = \sin(3t)\mathbf{i} + \cos(3t)\mathbf{j} + t\mathbf{k}$$

for $0 \le t \le 2\pi$. [AJN]

(a) Compute the principal unit normal vector $\mathbf{N}(t)$.

(b) Compute the curvature $\kappa(t)$.

$N(t) = \frac{T'(t)}{\|T'(t)\|}$ $\quad K = \frac{\|T'(t)\|}{\|r'(t)\|}$

$\mathbf{N}(t) = \boxed{\langle -\sin 3t, -\cos 3t, 0\rangle}$

$\kappa(t) = \boxed{9/10}$

$r'(t) = \langle 3\cos 3t, -3\sin 3t, 1\rangle$

$\|r'(t)\| = \sqrt{9\cos^2 3t + 9\sin^2 3t + 1} = \sqrt{9+1} = \sqrt{10}$

$T(t) = \langle \frac{3}{\sqrt{10}}\cos 3t, \frac{-3}{\sqrt{10}}\sin 3t, 1\rangle$

$T'(t) = \langle \frac{-9}{\sqrt{10}}\sin 3t, \frac{-9}{\sqrt{10}}\cos 3t, 0\rangle$

$\|T'(t)\| = \sqrt{\frac{81}{10}\sin^2 t + \frac{81}{10}\cos^2 t} = \sqrt{\frac{81}{10}} = \frac{9}{\sqrt{10}}$

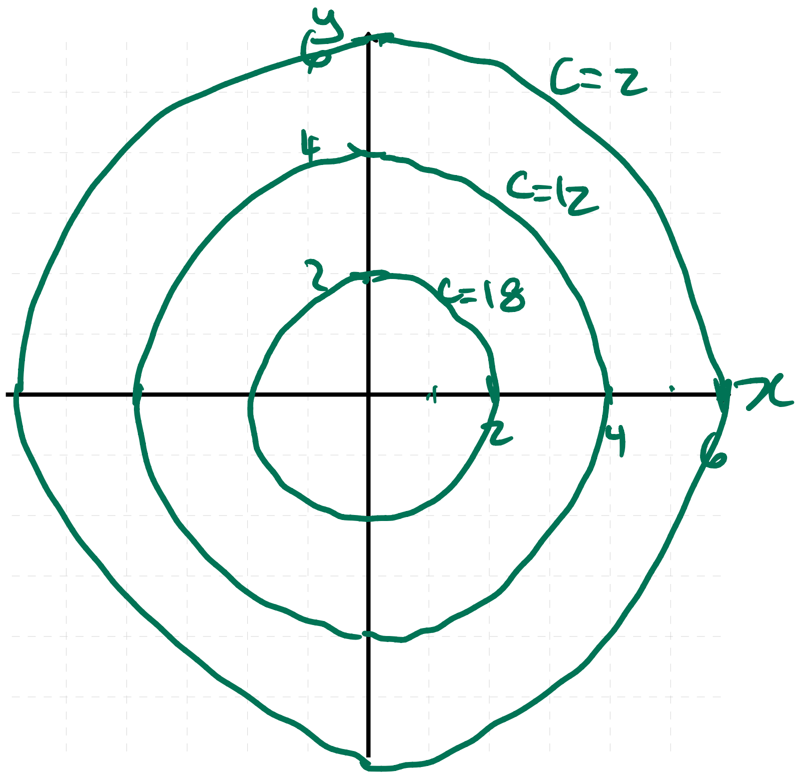
So $\quad N(t) = \langle -\sin 3t, -\cos 3t, 0\rangle$

and $\quad K = \frac{9/\sqrt{10}}{\sqrt{10}} = \frac{9}{10}$

5. (8 points) Draw a contour map on the axes provided including all three of the level curves $g(x, y) = c$ for the function

$$g(x, y) = 20 - \frac{1}{2}x^2 - \frac{1}{2}y^2, \quad c = 2, 12, 18.$$

Show your work for how you find the equation of each level set, include labels for the axes, and label each level set as well as ~~the~~ an x-intercept and y-intercept of each level set. [AJN]



$C=2$

$2 = 20 - \frac{1}{2}x^2 - \frac{1}{2}y^2$

$\Rightarrow \frac{1}{2}x^2 + \frac{1}{2}y^2 = 18$

$\Rightarrow x^2 + y^2 = 36$

$r = 6$

$C=12$

$12 = 20 - \frac{1}{2}x^2 - \frac{1}{2}y^2$

$\Rightarrow x^2 + y^2 = 16$

$r = 4$

$C=18$

$18 = 20 - \frac{1}{2}x^2 - \frac{1}{2}y^2$

$\Rightarrow x^2 + y^2 = 4$

$\Rightarrow r = 2$

6. (8 points) Show that the limit does not exist. To receive full credit, you must show work supporting your answer, use proper limit notation, and mention the test that you are using. [AJN]

$$\lim_{(x,y)\to(0,0)} \frac{xy}{|xy|}$$

along @ $y=x$

$$\lim_{(x,y)\to(0,0)} \frac{xy}{|xy|} = \lim_{(x,x)\to(0,0)} \frac{x^2}{|x^2|} = 1$$

along @ $y=-x$

$$\lim_{(x,y)\to(0,0)} \frac{xy}{|xy|} = \lim_{(x,-x)\to(0,0)} \frac{-x^2}{|-x^2|} = -1.$$

Since the limit has two different values as (x,y) approaches $(0,0)$ along two different paths,

$$\boxed{\text{by The TWO PATH TEST The limit is DNE}}$$

FORMULA SHEET

- $\langle u_1, u_2, u_3 \rangle \cdot \langle v_1, v_2, v_3 \rangle = u_1 v_1 + u_2 v_2 + u_3 v_3$
- $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}||\mathbf{v}| \cos(\theta)$
- $\langle u_1, u_2, u_3 \rangle \times \langle v_1, v_2, v_3 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$
- $|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}||\mathbf{v}||\sin(\theta)|$
- $L = \displaystyle\int_a^b |\mathbf{r}'(t)| \; dt$
- $s(t) = \displaystyle\int_{t_0}^t |\mathbf{r}'(\tau)| \; d\tau$
- $\mathbf{T} = \dfrac{\mathbf{v}}{|\mathbf{v}|} = \dfrac{d\mathbf{r}}{ds}$
- $\kappa = \left|\dfrac{d\mathbf{T}}{ds}\right| = \dfrac{1}{|\mathbf{v}|}\left|\dfrac{d\mathbf{T}}{dt}\right| = \dfrac{|\mathbf{v} \times \mathbf{a}|}{|\mathbf{v}|^3}$
- $\mathbf{N} = \dfrac{1}{\kappa}\dfrac{d\mathbf{T}}{ds} = \dfrac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|}$

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