

MATH 2550 G/J Midterm 1
VERSION B
Fall 2025
COVERS SECTIONS 12.1-12.6, 13.1-13.4, 14.1-14.2

Full name: _____ **GT ID:** _____

Honor code statement: I will abide strictly by the Georgia Tech honor code at all times. I will not use a calculator. **I do not have a phone within reach**, and I will not reference any website, application, or other CAS-enabled service. I will not consult with my notes or anyone during this exam. I will not provide aid to anyone else during this exam.

() All of the knowledge presented in this exam is entirely my own. I am initialing to the left to attest to my integrity.

Read all instructions carefully before beginning.

- Print your name and GT ID neatly above.
- You have 50 minutes to take the exam.
- You may not use aids of any kind.
- Please show your work [J] and annotate your work using proper notation [N].
- Good luck!

Question	Points
1	2
2	10
3	12
4	10
5	8
6	8
Total:	50

For T/F problems choose whether the statement is true or false. If the statement is *always* true, pick true. If the statement is *ever* false, pick false. Also please be sure to neatly fill in the bubble corresponding to your answer choice. [A]

1. (2 points) If $\mathbf{n} = \mathbf{u} \times \mathbf{v}$, for some vectors $\mathbf{u}, \mathbf{v}$ in $\mathbb{R}^3$, then $(\mathbf{u} \cdot \mathbf{n})^2 + (\mathbf{v} \cdot \mathbf{n})^2 = 0$.

☐ TRUE

☐ FALSE

2. (10 points) Let P be the plane defined by $x - 2y + 2z = 15$. Find (a) the vector equation for the line ℓ passing through the point $Q(2, 1, 3)$ which is orthogonal to P , and (b) find the intersection between this line ℓ and the plane P .

[AJN]

(a)

(b)

3. (12 points) Let $\mathbf{r}(t) = \langle \sin t - t \cos t, \cos t + t \sin t \rangle$, $2 \leq t \leq 5$. Find the curve's unit tangent vector $\mathbf{T}(t)$ and the length of the curve parametrized by $\mathbf{r}(t)$. [AJN]

$\mathbf{T}(t) =$

length is

4. (10 points) In this problem, you will work with the curve

$$\mathbf{r}(t) = t\mathbf{i} + (\ln \cos t)\mathbf{j}$$

for $-\pi/2 < t < \pi/2$.

[AJN]

(a) Compute the principal unit normal vector $\mathbf{N}(t)$.

(b) Compute the curvature $\kappa(t)$.

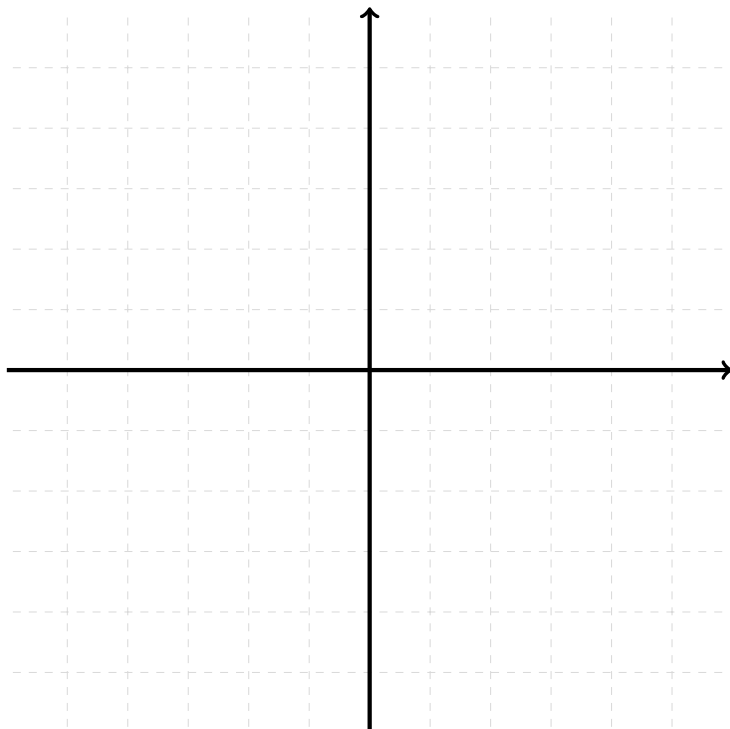
$\mathbf{N}(t) =$

$\kappa(t) =$

5. (8 points) Draw a contour map on the axes provided including all three of the level curves $g(x, y) = c$ for the function

$$g(x, y) = 4 - \frac{1}{3}x + \frac{1}{3}y^2, \quad c = 4, 5, 6.$$

Show your work for how you find the equation of each level set, include labels for the axes, and label each level set as well as any x -intercepts or y -intercepts of each level set. [AJN]



6. (8 points) Show that the limit does not exist. To receive full credit, you must show work supporting your answer, use proper limit notation, and mention the test that you are using.

[AJN]

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - y}{x - y}$$

FORMULA SHEET

- $\langle u_1, u_2, u_3 \rangle \cdot \langle v_1, v_2, v_3 \rangle = u_1 v_1 + u_2 v_2 + u_3 v_3$

- $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}| |\mathbf{v}| \cos(\theta)$

- $\langle u_1, u_2, u_3 \rangle \times \langle v_1, v_2, v_3 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$

- $|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}| |\mathbf{v}| |\sin(\theta)|$

- $L = \int_a^b |\mathbf{r}'(t)| \, dt$

- $s(t) = \int_{t_0}^t |\mathbf{r}'(\tau)| \, d\tau$

- $\mathbf{T} = \frac{\mathbf{v}}{|\mathbf{v}|} = \frac{d\mathbf{r}}{ds}$

- $\kappa = \left| \frac{d\mathbf{T}}{ds} \right| = \frac{1}{|\mathbf{v}|} \left| \frac{d\mathbf{T}}{dt} \right| = \frac{|\mathbf{v} \times \mathbf{a}|}{|\mathbf{v}|^3}$

- $\mathbf{N} = \frac{1}{\kappa} \frac{d\mathbf{T}}{ds} = \frac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|}$

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