MATH 2550 G/J Midterm 1
VERSION B
Fall 2025
COVERS SECTIONS 12.1-12.6, 13.1-13.4, 14.1-14.2

Full name: Key **GT ID:**

Honor code statement: I will abide strictly by the Georgia Tech honor code at all times. I will not use a calculator. **I do not have a phone within reach**, and I will not reference any website, application, or other CAS-enabled service. I will not consult with my notes or anyone during this exam. I will not provide aid to anyone else during this exam.

( ) All of the knowledge presented in this exam is entirely my own. I am initialing to the left to attest to my integrity.

Read all instructions carefully before beginning.

- Print your name and GT ID neatly above.
- You have 50 minutes to take the exam.
- You may not use aids of any kind.
- Please show your work [J] and annotate your work using proper notation [N].
- Good luck!

Question	Points
1	2
2	10
3	12
4	10
5	8
6	8
Total:	50

For T/F problems choose whether the statement is true or false. If the statement is *always* true, pick true. If the statement is *ever* false, pick false. Also please be sure to neatly fill in the bubble corresponding to your answer choice. [A]

1. (2 points) If $\mathbf{n} = \mathbf{u} \times \mathbf{v}$, for some vectors $\mathbf{u}, \mathbf{v}$ in $\mathbb{R}^3$, then $(\mathbf{u} \cdot \mathbf{n})^2 + (\mathbf{v} \cdot \mathbf{n})^2 = 0$.

● **TRUE** ○ **FALSE**

2. (10 points) Let P be the plane defined by $x - 2y + 2z = 15$. Find (a) the vector equation for the line ℓ passing through the point $Q(2, 1, 3)$ which is orthogonal to P, and (*b*) find the intersection between this line ℓ and the plane P.

[AJN]

(a) $\boxed{\ell(t) = \langle 2,1,3\rangle + t\langle 1,-2,2\rangle,\ t\in\mathbb{R}}$

$n = \langle 1,-2,2\rangle$ orthogonal to P.

$\boxed{\text{(b) } \langle 3,-1,5\rangle}$

(a) $\ell(t) = \langle 2,1,3\rangle + t\langle 1,-2,2\rangle,\ t\in\mathbb{R}$

$= \langle 2+t,\ 1-2t,\ 3+2t\rangle$

(b) Sub into plane eqn.

$x - 2y + 2z = 15$

$\Rightarrow (2+t) - 2(1-2t) + 2(3+2t) = 15$

$\Rightarrow \cancel{2} + t \cancel{-2} + 4t + 6 + 4t = 15$

$\Rightarrow 9t = 9 \Rightarrow t = 1$ intersection @ $t=1$

$\ell(1) = \langle 2,1,3\rangle + \langle 1,-2,2\rangle = \langle 3,-1,5\rangle$

3. (12 points) Let $\mathbf{r}(t) = \langle \sin t - t\cos t, \cos t + t\sin t\rangle$, $2 \le t \le 5$. Find the curve's unit tangent vector $\mathbf{T}(t)$ and the length of the curve parametrized by $\mathbf{r}(t)$. [AJN]

$\mathbf{T}(t) = \boxed{\langle \sin t, \cos t \rangle}$

length is $\boxed{21/2}$

$$r'(t) = \langle \cancel{\cos t} - \cancel{\cos t} + t\sin t, -\cancel{\sin t} + \cancel{\sin t} + t\cos t \rangle$$

$$= \langle t\sin t, t\cos t \rangle$$

$$\|r'(t)\| = \sqrt{t^2\sin^2 t + t^2\cos^2 t} = \sqrt{t^2} = t \quad (t \in [2,5])$$

$$\text{So } T(t) = \frac{r'(t)}{\|r'(t)\|} = \langle \sin t, \cos t \rangle$$

$$L = \int_2^5 \|r'(t)\|\,dt = \int_2^5 t\,dt = \frac{1}{2}t^2 \Big|_2^5$$

$$= \frac{25}{2} - \frac{4}{2} = \frac{21}{2}$$

4. (10 points) In this problem, you will work with the curve

$$\mathbf{r}(t) = t\mathbf{i} + (\ln\cos t)\mathbf{j}$$

for $-\pi/2 < t < \pi/2$. [AJN]

(a) Compute the principal unit normal vector $\mathbf{N}(t)$.

(b) Compute the curvature $\kappa(t)$.

$N(t) = \dfrac{T'(t)}{\|r'(t)\|}$ $\quad \kappa = \dfrac{\|T'(t)\|}{\|r'(t)\|}$

$\mathbf{N}(t) = \boxed{\langle -\sin t, -\cos t \rangle}$

$\kappa(t) = \boxed{\cos t}$

$r'(t) = \langle 1, \frac{1}{\cos t} \cdot -\sin t \rangle$

$= \langle 1, -\tan t \rangle$, $\|r'(t)\| = \sqrt{1 + \tan^2 t} = \sqrt{\sec^2 t} = \sec t$

So $T(t) = \langle \frac{1}{\sec t}, \frac{-\tan t}{\sec t} \rangle = \langle \cos t, -\sin t \rangle$.

$T'(t) = \langle -\sin t, -\cos t \rangle$, $\|T'(t)\| = \sqrt{\sin^2 t + \cos^2 t} = 1$

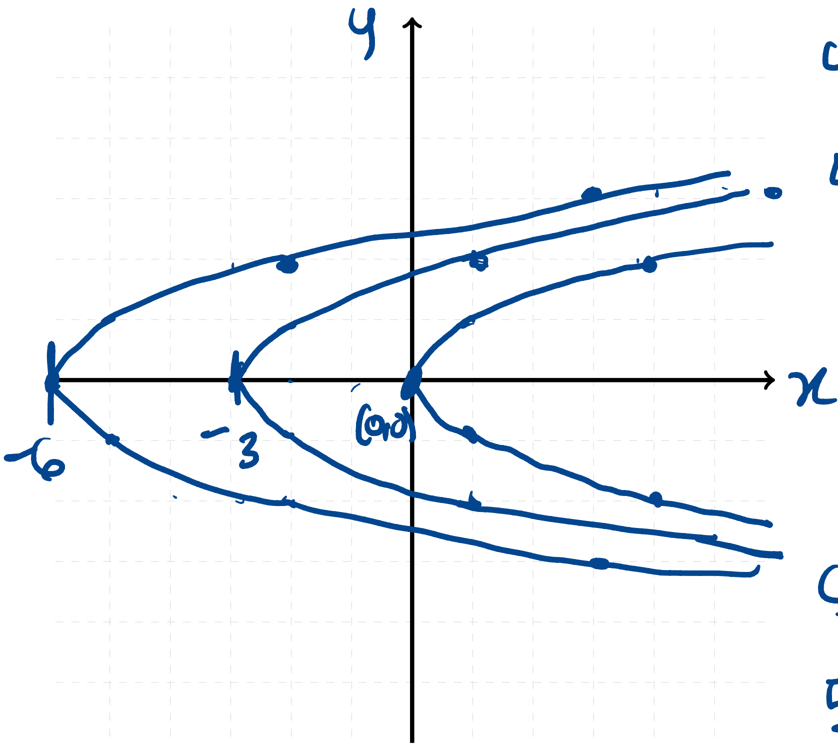
and $N(t) = \langle -\sin t, -\cos t \rangle$

$\kappa = \dfrac{1}{\sec t} = \cos t$

5. (8 points) Draw a contour map on the axes provided including all three of the level curves $g(x,y) = c$ for the function

$$g(x,y) = 4 - \frac{1}{3}x + \frac{1}{3}y^2, \quad c = 4, 5, 6.$$

Show your work for how you find the equation of each level set, include labels for the axes, and label each level set as well as an x-intercept and y-intercept of each level set. [AJN]



$\underline{c=4}$

$4 = 4 - \frac{1}{3}x + \frac{1}{3}y^2$

$\Rightarrow \quad x = y^2$

$\underline{c=5}$

$5 = 4 - \frac{1}{3}x + \frac{1}{3}y^2$

$\Rightarrow \quad \frac{1}{3}x = \frac{1}{3}y^2 - 1$

$\Rightarrow \quad x = y^2 - 3$

$\underline{c=6}$

$6 = 4 - \frac{1}{3}x + \frac{1}{3}y^2$

$\Rightarrow \quad \frac{1}{3}x = \frac{1}{3}y^2 - 2$

$\Rightarrow \quad x = y^2 - 6$

6. (8 points) Show that the limit does not exist. To receive full credit, you must show work supporting your answer, use proper limit notation, and mention the test that you are using. [AJN]

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y}{x-y}$$

along $x=0$

@ $x=0$

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y}{x-y} = \lim_{(0,y)\to(0,0)} \frac{-y}{-y} = 1.$$

along $y=0$

@ $y=0$

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y}{x-y} = \lim_{(x,0)\to(0,0)} \frac{x^2}{x}$$

$$= \lim_{(x,0)\to(0,0)} x = 0$$

$$\boxed{\text{by the TWO-PATH test the limit is DNE}}$$

FORMULA SHEET

- $\langle u_1, u_2, u_3\rangle \cdot \langle v_1, v_2, v_3\rangle = u_1v_1 + u_2v_2 + u_3v_3$
- $\mathbf{u}\cdot\mathbf{v} = |\mathbf{u}||\mathbf{v}|\cos(\theta)$
- $\langle u_1, u_2, u_3\rangle \times \langle v_1, v_2, v_3\rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$
- $|\mathbf{u}\times\mathbf{v}| = |\mathbf{u}||\mathbf{v}||\sin(\theta)|$
- $L = \displaystyle\int_a^b |\mathbf{r}'(t)|\ dt$
- $s(t) = \displaystyle\int_{t_0}^t |\mathbf{r}'(\tau)|\ d\tau$
- $\mathbf{T} = \dfrac{\mathbf{v}}{|\mathbf{v}|} = \dfrac{d\mathbf{r}}{ds}$
- $\kappa = \left|\dfrac{d\mathbf{T}}{ds}\right| = \dfrac{1}{|\mathbf{v}|}\left|\dfrac{d\mathbf{T}}{dt}\right| = \dfrac{|\mathbf{v}\times\mathbf{a}|}{|\mathbf{v}|^3}$
- $\mathbf{N} = \dfrac{1}{\kappa}\dfrac{d\mathbf{T}}{ds} = \dfrac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|}$

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