MATH 2550 G/J Midterm 2
VERSION B
Fall 2025
COVERS SECTIONS 14.3-14.8, 15.1-15.4

Full name: ______________________________ **GT ID:**____________

Honor code statement: I will abide strictly by the Georgia Tech honor code at all times. I will not use a calculator. I do not have a phone within reach, and I will not reference any website, application, or other CAS-enabled service. I will not consult with my notes or anyone during this exam. I will not provide aid to anyone else during this exam.

(   ) All of the knowledge presented in this exam is entirely my own. I am initialing to the left to attest to my integrity.

Read all instructions carefully before beginning.

- Print your name and GT ID neatly above.
- You have 50 minutes to take the exam.
- You may not use aids of any kind.
- Please show your work [J] and annotate your work using proper notation [N].
- Good luck!

Question	Points
1	2
2	4
3	6
4	6
5	6
6	10
7	8
8	8
Total:	50

For problems 1-2 choose whether each statement is true or false. If the statement is *always* true, pick true. If the statement is *ever* false, pick false. For all problems on this page please be sure to neatly fill in the bubble corresponding to your answer choice. [A]

1. (2 points) If $f(x, y)$ is continuous on $R = [1, 2] \times [3, 4]$ then the value of $\int_1^2 \int_3^4 f(x, y)\, dy\, dx$ equals the value of $\int_3^4 \int_1^2 f(x, y)\, dy\, dx$.

   ◯ **TRUE** ◯ **FALSE**

2. (4 points) Compute f_{xx} and f_{yx} for the function $f(x, y) = \sin(xy) + ye^{3x}$. [AJN]

   $f_{xx} = \boxed{}$

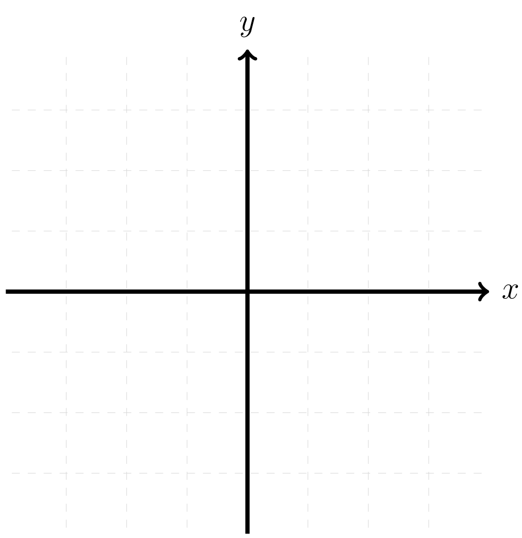
   $f_{yx} = \boxed{}$

3. (6 points) Compute Df given $f(r,s,t) = \begin{bmatrix} s^2t + r/s \\ 3r - 4s + 5t \end{bmatrix}$. [AJN]

$Df=$

4. (6 points) Sketch R the region of integration and convert the given integral to polar coordinates. *Do not evaluate!* [AN]

$$\int_0^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} x \; dy \; dx$$



Volume=

5. (6 points) Use the function $z = f(x, y)$ and the point P to answer the questions below. [AJN]

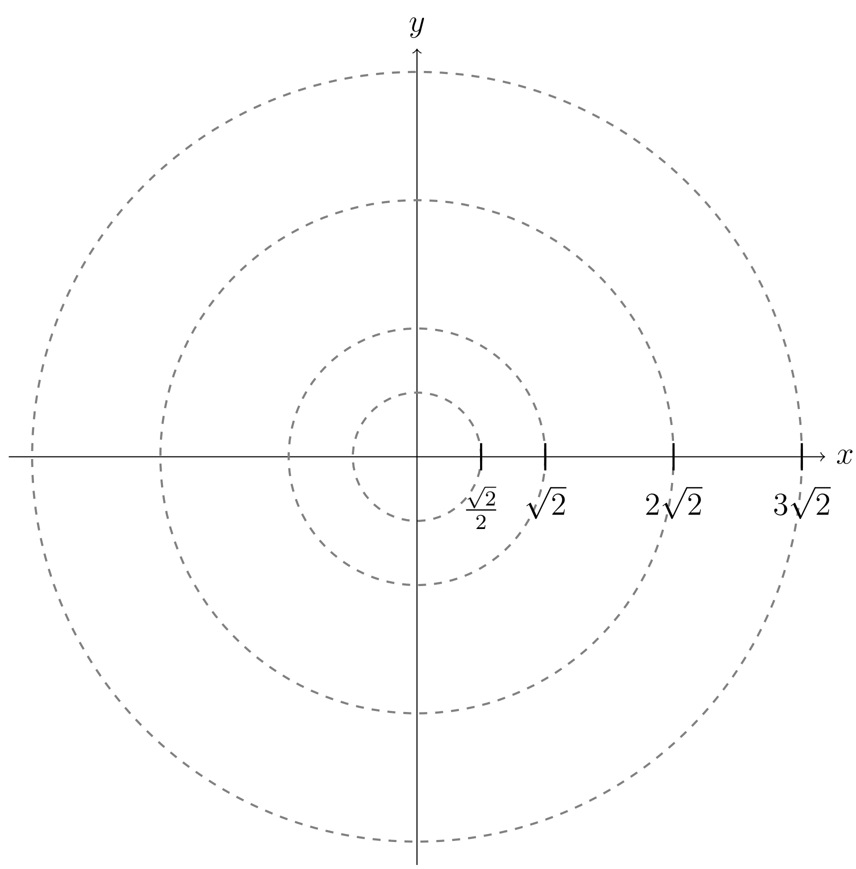
$$f(x, y) = \ln(x^2 + y^2) \text{ and } P(-1, 1)$$

(a) Find the gradient ∇f of f.

(b) Find the directional derivative $D_{\mathbf{u}}f$ of f in the direction of $\mathbf{u} = \langle 4, 3 \rangle$ at $P(-1, 1)$.

(c) Find a unit vector $\mathbf{v}$ which points in the direction which *maximizes* the value of $D_{\mathbf{u}}f$ at $P(-1, 1)$.

(d) Sketch $P(-1, 1)$ and the gradient vector $\nabla f(P)$ on the axes provided.

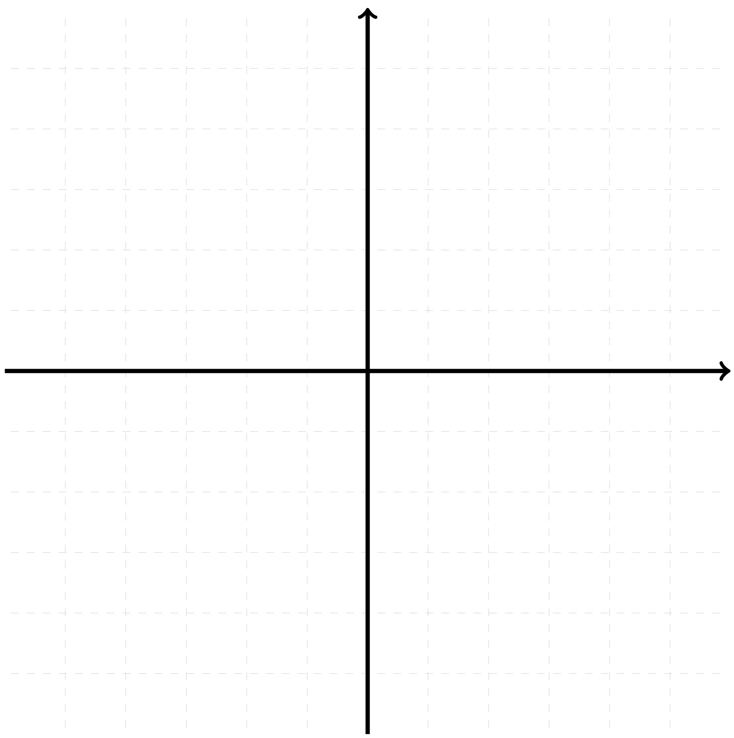


6. (10 points) Evaluate $\frac{\partial W}{\partial t}$ at the point $(s,t) = (2, \frac{\pi}{4})$. [AJN]

$$W = xy + \ln y,\ x = s + t,\ y = \cos t.$$

7. (8 points) Sketch the region R including all labels for each of the axes, each boundary curve, and each intersection point between curves and axes. Then, compute the average value of $f(x, y) = xy$ over the region R in the first quadrant bounded by the lines $x = 0$, $y = 0$, $y = \frac{1}{2}x$, and $y = 2$.

[AJN]



8. (8 points) Find all critical points and use the Hessian matrix to classify the critical points of the function. [AJN]

$$f(x, y) = x^2 + xy + 3x + 2y + 5$$

FORMULA SHEET

- Total Derivative: For $\mathbf{f}(x_1, \ldots, x_n) = \langle f_1(x_1, \ldots, x_n), \ldots, f_m(x_1, \ldots, x_n)\rangle$

$$D\mathbf{f} = \begin{bmatrix} (f_1)_{x_1} & (f_1)_{x_2} & \cdots & (f_1)_{x_n} \\ (f_2)_{x_1} & (f_2)_{x_2} & \cdots & (f_2)_{x_n} \\ \vdots & \ddots & \cdots & \vdots \\ (f_m)_{x_1} & (f_m)_{x_2} & \cdots & (f_m)_{x_n} \end{bmatrix}$$

- Linearization: Near $\mathbf{a}$, $f(\mathbf{x}) \approx L(\mathbf{x}) = f(\mathbf{a}) + Df(\mathbf{a})(\mathbf{x} - \mathbf{a})$

- Chain Rule: If $h = g(f(\mathbf{x}))$ then $Dh(\mathbf{x}) = Dg(f(\mathbf{x}))Df(\mathbf{x})$

- Implicit Differentiation: If z is implicitly given in terms of x and y by $F(x, y, z) = c$, then $\dfrac{\partial z}{\partial x} = \dfrac{-F_x}{F_z}$ and $\dfrac{\partial z}{\partial y} = \dfrac{-F_y}{F_z}$.

- Directional Derivative: If $\mathbf{u}$ is a unit vector, $D_{\mathbf{u}}f(P) = Df(P)\mathbf{u} = \nabla f(P) \cdot \mathbf{u}$

- The tangent line to a level curve of $f(x, y)$ at (a, b) is $0 = \nabla f(a, b) \cdot \langle x - a, y - b\rangle$

- The tangent plane to a level surface of $f(x, y, z)$ at (a, b, c) is

$$0 = \nabla f(a, b, c) \cdot \langle x - a, y - b, z - c\rangle.$$

- Hessian Matrix: For $f(x, y)$, $Hf(x, y) = \begin{bmatrix} f_{xx} & f_{yx} \\ f_{xy} & f_{yy} \end{bmatrix}$

- Second Derivative Test: If (a, b) is a critical point of $f(x, y)$ then
  1. If $\det(Hf(a, b)) > 0$ and $f_{xx}(a, b) < 0$ then f has a local maximum at (a, b)
  2. If $\det(Hf(a, b)) > 0$ and $f_{xx}(a, b) > 0$ then f has a local minimum at (a, b)
  3. If $\det(Hf(a, b)) < 0$ then f has a saddle point at (a, b)
  4. If $\det(Hf(a, b)) = 0$ the test is inconclusive

- Area/volume: $\text{area}(R) = \iint_R dA$

- Trig identities: $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$, $\quad \cos^2(x) = \frac{1}{2}(1 + \cos(2x))$

- Average value: $f_{avg} = \dfrac{\iint_R f(x, y)dA}{\text{area of } R}$

- Polar coordinates: $x = r\cos(\theta)$, $\quad y = r\sin(\theta)$, $\quad dA = r\ dr\ d\theta$

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