MATH 2550 G/J Midterm 2 Make-up
VERSION C
Fall 2025
COVERS SECTIONS 14.3-14.8, 15.1-15.4

Full name: ______________________________ **GT ID:** ____________

Honor code statement: I will abide strictly by the Georgia Tech honor code at all times. I will not use a calculator. I do not have a phone within reach, and I will not reference any website, application, or other CAS-enabled service. I will not consult with my notes or anyone during this exam. I will not provide aid to anyone else during this exam.

(   ) All of the knowledge presented in this exam is entirely my own. I am initialing to the left to attest to my integrity.

Read all instructions carefully before beginning.

- Print your name and GT ID neatly above.
- You have 50 minutes to take the exam.
- You may not use aids of any kind.
- Please show your work [J] and annotate your work using proper notation [N].
- Good luck!

Question	Points
1	2
2	6
3	7
4	7
5	8
6	10
7	10
Total:	50

For problems 1-2 choose whether each statement is true or false. If the statement is *always* true, pick true. If the statement is *ever* false, pick false. For all problems on this page please be sure to neatly fill in the bubble corresponding to your answer choice. [A]

1. (2 points) If $f(x,y) = 1$ for all (x,y) in $R = [3,6] \times [1,4]$, then the value of $\int_1^4 \int_3^6 f(x,y)\,dy\,dx$ equals the value of $\int_3^6 \int_1^4 f(x,y)\,dy\,dx$.

   ◯ **TRUE** ◯ **FALSE**

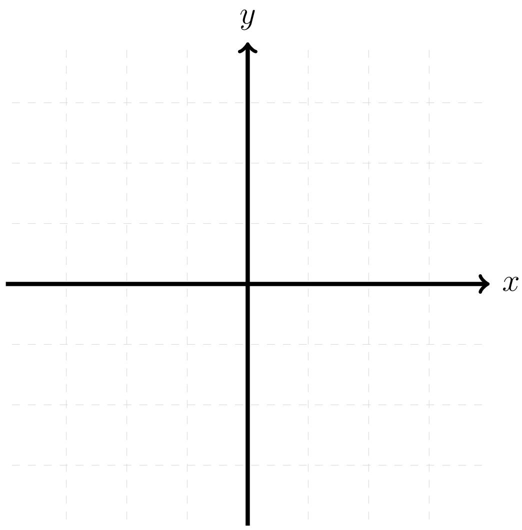
2. (6 points) Compute f_{xx} and f_{yx} for the function $f(x,y) = x\sec(y) + \frac{y}{x}$. [AJN]

$f_{xx} =$

$f_{yx} =$

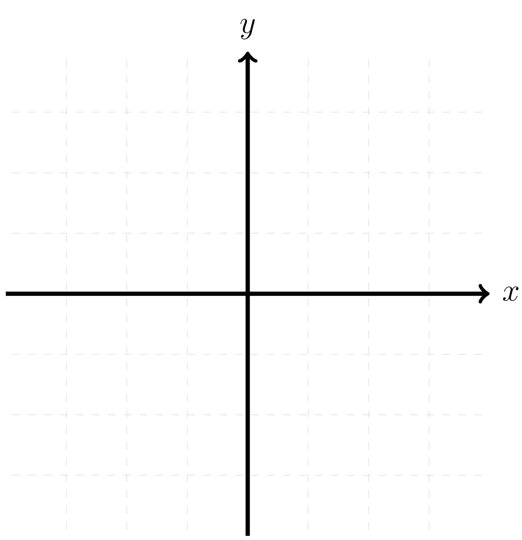
3. (7 points) Sketch R the region of integration and switch the order of integration. That is, rewrite the integral in the order $dx\ dy$. *Do not evaluate!* [AN]

$$\int_0^1 \int_0^{3x} y\ dy\ dx$$



4. (7 points) Sketch R the region of integration and set up an integral in polar coordinates for evaluating $\iint_R f(x,y)\ dA$. Note that R is the region inside the circle of radius $r = 3$ which is under the line $y = x$. *Do not evaluate!* [AN]

$$R = \left\{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 9,\ y \leq x\right\},\ \text{and}\ f(x,y) = xy.$$



Volume=

5. (8 points) Use the function $z = f(x, y)$ and the point P to answer the questions below. [AJN]

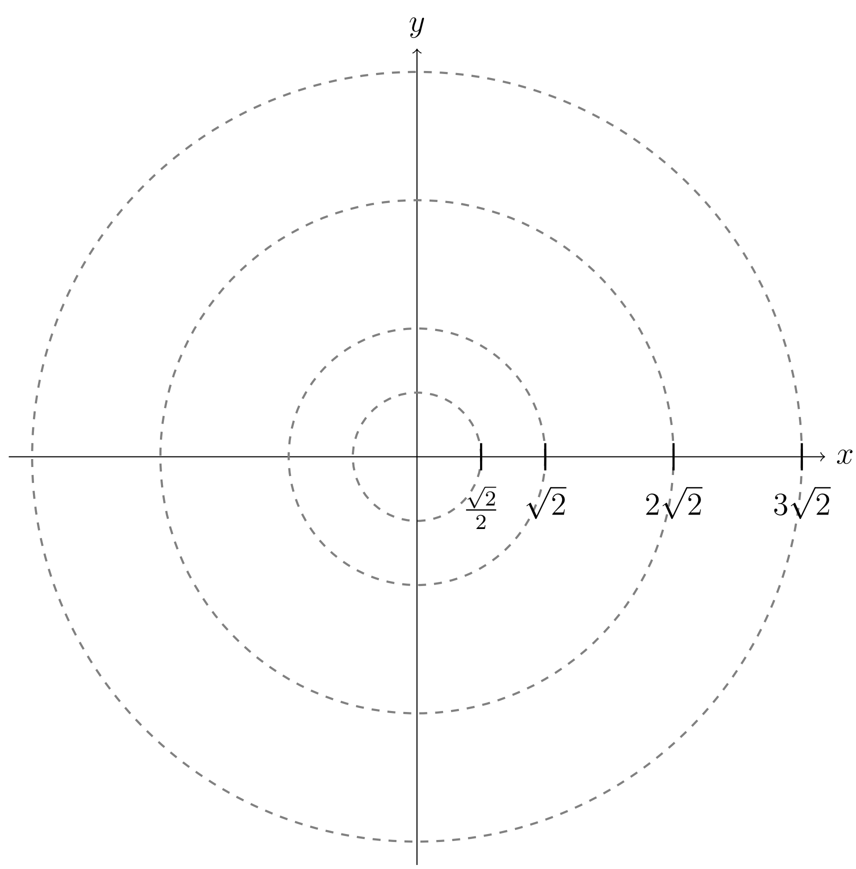
$$f(x, y) = 2\ln(x^2 + y^2) \text{ and } P(2, -2)$$

(a) Find the gradient ∇f of f.

(b) Find the directional derivative $D_{\mathbf{u}}f$ of f in the direction of $\mathbf{u} = \langle 1, 3\rangle$ at $P(2, -2)$.

(c) Find a unit vector $\mathbf{v}$ which points in the direction which *maximizes* the value of $D_{\mathbf{u}}f$ at $P(2, -2)$.

(d) Sketch $P(2, -2)$ and the gradient vector $\nabla f(P)$ on the axes provided.

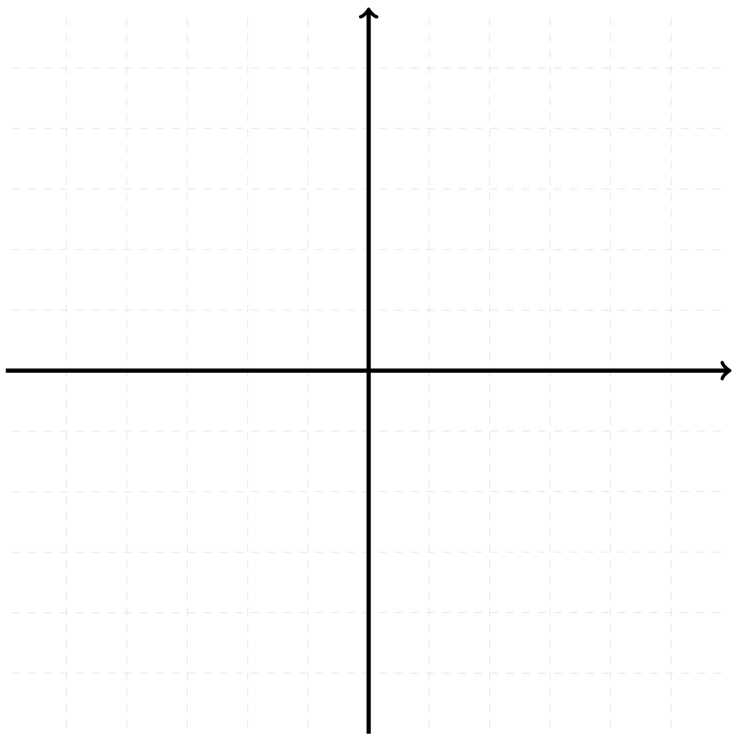


6. (10 points) Evaluate $\frac{\partial W}{\partial t}$ at the point $(s, t) = (1, 1)$. [AJN]

$$W = xy + e^{3x},\ x = s + 2t,\ y = \ln 3t.$$

7. (10 points) Sketch the domain R including all labels on the axes provided, and find the absolute maxima and minima of the function $f(x, y) = x^2 + 2x + y^2 + 2y$ on R.

$R = \left\{(x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq 2, -2 \leq y \leq 2\right\}$ is the rectangle with vertices $(0, \pm 2)$, $(2, \pm 2)$.



FORMULA SHEET

- Total Derivative: For $\mathbf{f}(x_1, \ldots, x_n) = \langle f_1(x_1, \ldots, x_n), \ldots, f_m(x_1, \ldots, x_n)\rangle$

$$D\mathbf{f} = \begin{bmatrix} (f_1)_{x_1} & (f_1)_{x_2} & \cdots & (f_1)_{x_n} \\ (f_2)_{x_1} & (f_2)_{x_2} & \cdots & (f_2)_{x_n} \\ \vdots & \ddots & \cdots & \vdots \\ (f_m)_{x_1} & (f_m)_{x_2} & \cdots & (f_m)_{x_n} \end{bmatrix}$$

- Linearization: Near $\mathbf{a}$, $f(\mathbf{x}) \approx L(\mathbf{x}) = f(\mathbf{a}) + Df(\mathbf{a})(\mathbf{x} - \mathbf{a})$

- Chain Rule: If $h = g(f(\mathbf{x}))$ then $Dh(\mathbf{x}) = Dg(f(\mathbf{x}))Df(\mathbf{x})$

- Implicit Differentiation: If z is implicitly given in terms of x and y by $F(x, y, z) = c$, then $\frac{\partial z}{\partial x} = \frac{-F_x}{F_z}$ and $\frac{\partial z}{\partial y} = \frac{-F_y}{F_z}$.

- Directional Derivative: If $\mathbf{u}$ is a unit vector, $D_{\mathbf{u}}f(P) = Df(P)\mathbf{u} = \nabla f(P) \cdot \mathbf{u}$

- The tangent line to a level curve of $f(x, y)$ at (a, b) is $0 = \nabla f(a, b) \cdot \langle x - a, y - b\rangle$

- The tangent plane to a level surface of $f(x, y, z)$ at (a, b, c) is

$$0 = \nabla f(a, b, c) \cdot \langle x - a, y - b, z - c\rangle.$$

- Hessian Matrix: For $f(x, y)$, $Hf(x, y) = \begin{bmatrix} f_{xx} & f_{yx} \\ f_{xy} & f_{yy} \end{bmatrix}$

- Second Derivative Test: If (a, b) is a critical point of $f(x, y)$ then
  1. If $\det(Hf(a, b)) > 0$ and $f_{xx}(a, b) < 0$ then f has a local maximum at (a, b)
  2. If $\det(Hf(a, b)) > 0$ and $f_{xx}(a, b) > 0$ then f has a local minimum at (a, b)
  3. If $\det(Hf(a, b)) < 0$ then f has a saddle point at (a, b)
  4. If $\det(Hf(a, b)) = 0$ the test is inconclusive

- Area/volume: $\text{area}(R) = \iint_R dA$

- Trig identities: $\sin^2(x) = \frac{1}{2}(1 - \cos(2x)), \quad \cos^2(x) = \frac{1}{2}(1 + \cos(2x))$

- Average value: $f_{avg} = \dfrac{\iint_R f(x, y)dA}{\text{area of } R}$

- Polar coordinates: $x = r\cos(\theta), \quad y = r\sin(\theta), \quad dA = r\ dr\ d\theta$

SCRATCH PAPER - PAGE LEFT BLANK