

Example 22. *You try it!* Find (a) an arc length parameterization $s(t)$ of the curve \mathcal{C} , the portion of the helix of radius 4 in \mathbb{R}^3 parameterized by $\mathbf{r}(t) = \langle 4 \cos(t), 4 \sin(t), 3t \rangle, 0 \leq t \leq \pi/2$, and (b) use $s(t)$ to find L the length of \mathcal{C}

§13.3 & 13.4 - Curvature, Tangents, Normals

The next idea we are going to explore is the curvature of a curve in space along with two vectors that orient the curve.

First, we need the **unit tangent vector**, denoted **T**:

- In terms of an arc-length parameter s : _____
- In terms of any parameter t : _____

This lets us define the **curvature**, $\kappa(s) =$ _____

Example 23. In Example 21 we found an arc length parameterization of the circle of radius 4 centered at $(0, 0)$ in \mathbb{R}^2 :

$$\mathbf{r}(s) = \left\langle 4 \cos \left(\frac{s}{4} \right), 4 \sin \left(\frac{s}{4} \right) \right\rangle, \quad 0 \leq s \leq 8\pi.$$

Use this to find $\mathbf{T}(s)$ and $\kappa(s)$.

Question: In which direction is \mathbf{T} changing?

This is the direction of the **principal unit normal**, $\mathbf{N}(s) =$ _____

We said last time that it is often hard to find arc length parameterizations, so what do we do if we have a generic parameterization $\mathbf{r}(t)$?

• $\mathbf{T}(t) =$ _____

• $\mathbf{N}(t) =$ _____

• $\kappa(t) =$ _____ or _____

Example 24. Find $\mathbf{T}, \mathbf{N}, \kappa$ for the helix $\mathbf{r}(t) = \langle 2 \cos(t), 2 \sin(t), t - 1 \rangle$, $t \in \mathbb{R}$.

Example 25. *You try it!* Find $\mathbf{T}, \mathbf{N}, \kappa$ for the curve parametrized by

$$\mathbf{r}(t) = (\cos t + t \sin t)\mathbf{i} + (\sin t - t \cos t)\mathbf{j} + 3\mathbf{k}, \quad t \in \mathbb{R}.$$

§14.1 Functions of Multiple Variables

Definition 26. A _____ is a rule that assigns to each _____ of real numbers (x, y) in a set D a _____ denoted by $f(x, y)$.

$$f : D \rightarrow \mathbb{R}, \text{ where } D \subseteq \mathbb{R}^2$$

Example 27. Three examples are

$$f(x, y) = x^2 + y^2, \quad g(x, y) = \ln(x + y), \quad h(x, y) = \frac{1}{\sqrt{x + y}}.$$

Example 28. Find the largest possible domains of f, g , and h .

Definition 29. If f is a function of two variables with domain D , then the graph of f is the set of all points (x, y, z) in \mathbb{R}^3 such that $z = f(x, y)$ and (x, y) is in D .

Here are the graphs of the three functions above.

Example 30. Suppose a small hill has height $h(x, y) = 4 - \frac{1}{4}x^2 - \frac{1}{4}y^2$ m at each point (x, y) . How could we draw a picture that represents the hill in 2D?

Definition 31. The _____ (also called _____) of a function f of two variables are the curves with equations _____, where k is a constant (in the range of f). A plot of _____ for various values of z is a _____ (or _____).

Some common examples of these are:

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-
-

Example 32. Create a contour diagram of $f(x, y) = x^2 - y^2$

Definition 33. The _____ of a surface are the curves of _____ of the surface with planes parallel to the _____.

Example 34. Use the traces and contours of $z = f(x, y) = 4 - 2x - y^2$ to sketch the portion of its graph in the first octant.

Let's check our work: <https://tinyurl.com/math2551-2var-graph>

Definition 35. A _____ is a rule that assigns to each _____ of real numbers (x, y, z) in a set D a _____ denoted by $f(x, y, z)$.

$$f : D \rightarrow \mathbb{R}, \text{ where } D \subseteq \mathbb{R}^3$$

We can still think about the domain and range of these functions. Instead of level curves, we get level surfaces.

Example 36. Describe the largest possible domain of the function

$$f(x, y, z) = \frac{1}{4 - x^2 - y^2 - z^2}.$$

Example 37. Describe the level surfaces of the function $g(x, y, z) = 2x^2 + y^2 + z^2$.