Quiz 1

Be sure to follow the quiz instructions in order to avoid a deduction in points. Submissions are due in Gradescope by 11:59pm on Friday; no late work is accepted.

Name: Key

Question #1: Find the area of the parallelogram determined by the points by using the cross product.

$$P(1, 2, 3),\ Q(2, 4, 5),\ R(3, 0, 1)$$

$$\vec{PQ} = \langle 2,4,5\rangle - \langle 1,2,3\rangle = \langle 1,2,2\rangle$$

$$\vec{PR} = \langle 3,0,1\rangle - \langle 1,2,3\rangle = \langle 2,-2,-2\rangle$$

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 2 & -2 & -2 \end{vmatrix} = \hat{i}(-4+4) - \hat{j}(-2-4) + \hat{k}(-2-4)$$

$$= \langle 0, 6, -6\rangle$$

Area of parallelogram is

$$\|\vec{PQ} \times \vec{PR}\| = \sqrt{0^2+6^2+6^2} = \sqrt{72} = \boxed{6\sqrt{2}}$$

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Question #2: Let P be the plane defined by $x+2y+z=7$. Find (a) the vector equation for the line ℓ passing through the point $Q(1,1,1)$ which is orthogonal to P, and (b) find the intersection between this line ℓ and the plane P.

$\vec{n} = \langle 1,2,1 \rangle$ is orthogonal to P plane defined by

$$x+2y+z=7.$$

line equation is $\ell(t) = \vec{OQ} + t\vec{n}$, $t \in \mathbb{R}$

(a) $\boxed{\ell(t) = \langle 1,1,1 \rangle + t\langle 1,2,1 \rangle, \ t \in \mathbb{R}}$

to find the intersection with plane just substitute

$\ell(t) = \langle x,y,z \rangle = \langle 1+t, 1+2t, 1+t \rangle$ into plane eqn.

$$(1+t) + 2(1+2t) + (1+t) = 7$$

$$\Rightarrow 4 + 6t = 7 \Rightarrow 6t = 3 \Rightarrow t = 1/2$$

So $\ell(1/2) = \langle 1+1/2, 1+2(1/2), 1+1/2 \rangle = \langle 3/2, 2, 3/2 \rangle$

(b) $\boxed{(3/2, 2, 3/2)}$

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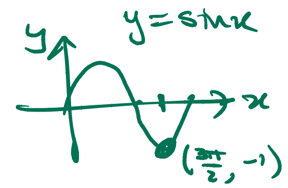
Question #3: Suppose a particle is traveling and it's location at time t, $0 \le t \le 2\pi$, is given by the parametrization $\mathbf{r}(t) = \langle \sin(3t), \cos(3t), t \rangle$, with t measured in seconds and $\mathbf{r}(t)$ in meters.

(a) Find the speed of the particle one quarter of the way through it's journey, at time $t = \pi/2$.

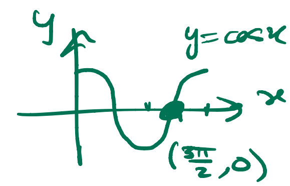
(b) Find a unit vector which points in the direction of travel at time $t = \pi/2$.

(c) Find the position of the particle halfway through the journey, at time $t = \pi$.

(a) Speed at time t is $\|r'(t)\|$



$r'(t) = \langle 3\cos 3t, -3\sin 3t, 1 \rangle$



So $\|r'(t)\| = \sqrt{\underbrace{9\cos^2 3t + 9\sin^2 3t}_{9} + 1} = \boxed{\sqrt{10}}$

(b) $\dfrac{r'(\pi/2)}{\|r'(\pi/2)\|} = \dfrac{1}{\sqrt{10}} \left\langle 3\cos\left(\frac{3\pi}{2}\right), -3\sin\left(\frac{3\pi}{2}\right), 1 \right\rangle$

$= \boxed{\left\langle 0, \frac{+3}{\sqrt{10}}, \frac{1}{\sqrt{10}} \right\rangle}$

(c) $r(\pi) = \langle \sin(3\pi), \cos(3\pi), \pi \rangle = \boxed{\langle 0, -1, \pi \rangle}$