Quiz 2

Be sure to follow the quiz instructions in order to avoid a deduction in points. Submissions are due in Gradescope by 11:59pm on Friday; no late work is accepted.

Name: *Key*

Question #1: Find the value of the limit.

$$\lim_{(x,y)\to(9,4)} \frac{\sqrt{x}-\sqrt{y+5}}{x-y-5}$$

$$\lim_{(x,y)\to(9,4)} \frac{\sqrt{x}-\sqrt{y+5}}{x-y-5} = \lim_{(x,y)\to(9,4)} \frac{\sqrt{x}-\sqrt{y+5}}{x-y-5} * \frac{\sqrt{x}+\sqrt{y+5}}{\sqrt{x}+\sqrt{y+5}}$$

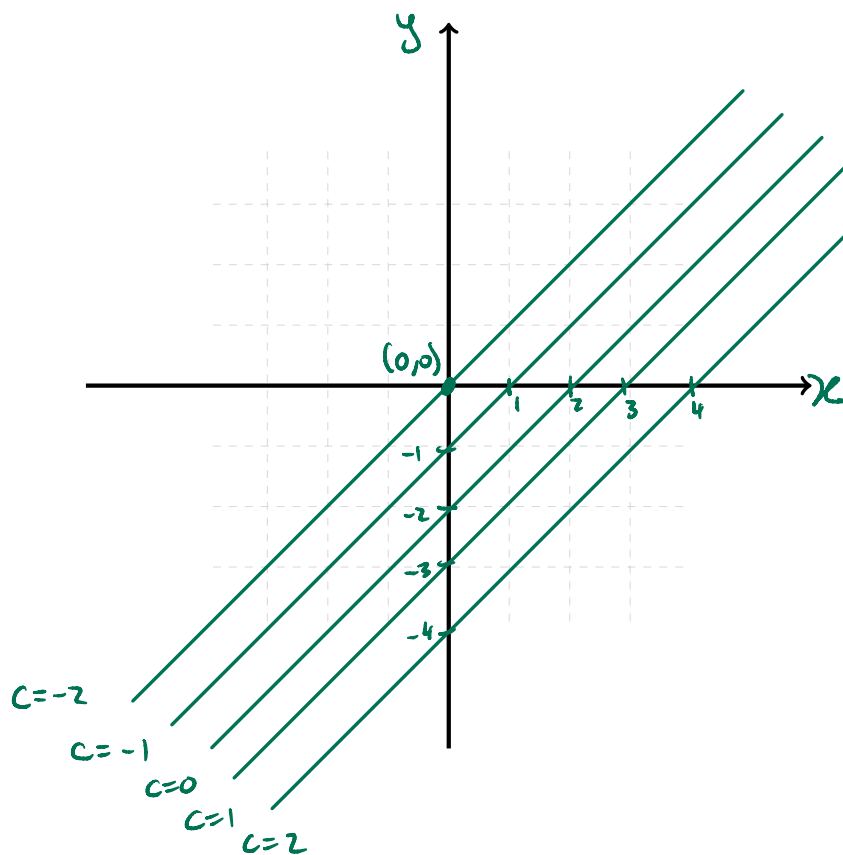$$= \lim_{(x,y)\to(9,4)} \frac{x-(y+5)}{(x-y-5)(\sqrt{x}+\sqrt{y+5})} = \lim_{(x,y)\to(9,4)} \frac{1}{\sqrt{x}+\sqrt{y+5}}$$

$$= \frac{1}{\sqrt{9}+\sqrt{4+5}} = \frac{1}{3+3} = \boxed{\frac{1}{6}}$$

Question #2: Draw a contour map on the axes provided including all five of the level curves $g(x,y) = c$ for the function $g(x,y) = x - y - 2$, $c = -2, -1, 0, 1, 2$. For full credit show your work for how you find the equation of each level set, include labels for the axes, and label each level set as well as the x-intercept and y-intercept of each level set.

@ $c = -2$

$x - y - 2 = -2$

$\rightarrow \quad x - y = 0 \Rightarrow y = x$

@ $c = -1$

$x - y - 2 = -1$

$\Rightarrow y = x - 1$

@ $c = 0$

$x - y - 2 = 0 \Rightarrow y = x - 2$

@ $c = 1$

$x - y - 2 = 1 \Rightarrow y = x - 3$

@ $c = -2$

$x - y - 2 = -2 \Rightarrow y = x - 4$

Question #3: Show that the limit does not exist by using the Two-Path Test.

$$\lim_{(x,y)\to(0,0)} \frac{x^3 y}{x^6 + y^2}$$

along x-axis

@ $y = 0$

$$\lim_{(x,y)\to(0,0)} \frac{x^3 y}{x^6 + y^2} = \lim_{(x,y)\to(0,0)} \frac{x^3 \cdot 0}{x^6 + y^2} = \lim_{x\to 0} \frac{0}{x^6} = 0 \checkmark$$

along $y = x^3$

@ $y = x^3$

$$\lim_{(x,y)\to(0,0)} \frac{x^3 y}{x^6 + y^2} = \lim_{(x,x^3)\to(0,0)} \frac{x^3 x^3}{x^6 + (x^3)^2} = \lim_{x\to 0} \frac{x^6}{2x^6} = \frac{1}{2} \checkmark$$

Since the limit along different paths is
a different value,

The limit is DNE by the
two path test