Quiz 3

Be sure to follow the quiz instructions in order to avoid a deduction in points. Submissions are due in Gradescope by 11:59pm on Friday; no late work is accepted.

Name: Key

Question #1: Let $f : \mathbb{R}^3 \to \mathbb{R}$ and $\mathbf{r}(s,t) : \mathbb{R}^2 \to \mathbb{R}^3$. (a) Give the general formula for $\frac{\partial f}{\partial s}$. Next, suppose f is the function $f(x,y,z) = x^2y + sinz$ and r is the function

$$\mathbf{r}(s,t) = \begin{bmatrix} s+1 \\ s^2t \\ \ln(1+st) \end{bmatrix}.$$

and (b) use the Chain Rule to evaluate $D(f(\mathbf{r}(s,t)))|_{(s,t)=(1,0)}$.
Note: you do not need [J] for part (a).

(a) $\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial s}$

(b)

$Df = \begin{bmatrix} 2xy & x^2 & \cos z \end{bmatrix}$ and $Dr = \begin{bmatrix} 1 & 0 \\ 2st & s^2 \\ \frac{t}{1+st} & \frac{s}{1+st} \end{bmatrix}$

@ $(s,t) = (1,0)$

$Dr|_{(1,0)} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$

@ $(s,t) = (1,0)$ then $\langle x,y,z \rangle = r(1,0) = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$

So $Df|_{(2,0,0)} = \begin{bmatrix} 0 & 4 & 1 \end{bmatrix}$

So $(Df * Dr)|_{(s,t)=(1,0)} = \begin{bmatrix} 0 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 5 \end{bmatrix}$

So $Df * Dr|_{(1,0)} = \begin{bmatrix} 0 & 5 \end{bmatrix}$

Name: Key Quiz 1

Question #2: Use the Implicit Differentiation formula from lecture to compute $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for the cone $x^2 + y^2 = z^2$.

Formula: $\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}$ and $\frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$

Surface implicitly defined by

$$F(x,y) = x^2 + y^2 - z^2 = 0$$

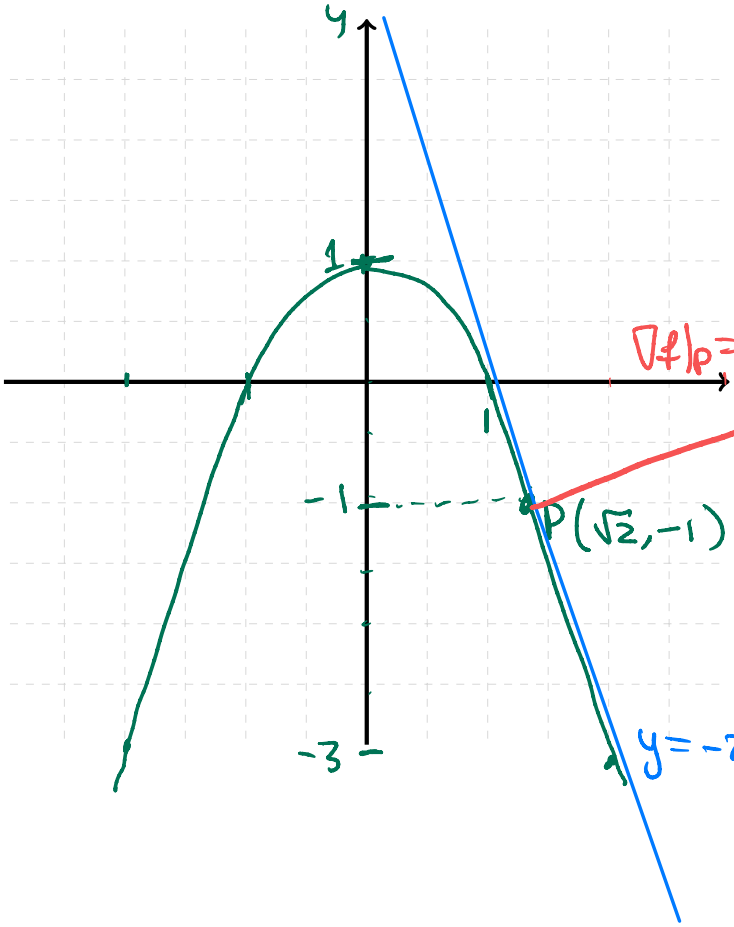
$\frac{\partial F}{\partial x} = 2x$, $\frac{\partial F}{\partial y} = 2y$, $\frac{\partial F}{\partial z} = -2z$

So $\frac{\partial z}{\partial x} = \frac{-2x}{(-2z)} = \boxed{\frac{x}{z}}$

and $\frac{\partial z}{\partial y} = \frac{-2y}{(-2z)} = \boxed{\frac{y}{z}}$

Name: Key Quiz 1

Question #3: Sketch the curve $x^2+y=1$ together with (a) the vector $\nabla f|_P$ and (b) the tangent line at $P(\sqrt{2},-1)$. Be sure to label the axes, the coordinates of the point P and the gradient vector $\nabla f|_P$, as well as the tangent line with the equation that defines it.



curve $f(x,y) = x^2+y-1=0$

$\nabla f = [f_x \; f_y]^T$

$= [2x \; 1]^T$

$$\nabla f\Big|_{(\sqrt{2},1)} = \begin{bmatrix} 2\sqrt{2} \\ 1 \end{bmatrix}$$

$2\sqrt{2} \approx 2.8$

tangent line eqn.

$f_x(P)(x-x_0) + f_y(P)(y-y_0) = 0$

So substituting

$2\sqrt{2}(x-\sqrt{2}) + 1(y+1) = 0$

$\Rightarrow \; y = -2\sqrt{2}x + 4 - 1$

$\Rightarrow \; \boxed{y = -2\sqrt{2}x + 3}$